Image-Subtraction Photometry of the Globular Cluster M3: identification of new double-mode RR Lyrae stars

Gisella Clementini

INAF - Osservatorio Astronomico di Bologna, Via Ranzani 1, I-40127 Bologna, Italy. email: gisella@sean.bo.astro.it

T. Michael Corwin[1]

Department of Physics, University of North Carolina at Charlotte, Charlotte, NC 28223. email: mcorwin@uncc.edu

Bruce W. Carney[1]

Department of Physics and Astronomy, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599. email: bruce@astro.unc.edu

Andrew N. Sumerel

Department of Physics, University of North Carolina at Charlotte, Charlotte, NC 28223. email: ansumere@uncc.edu

ABSTRACT

We have applied the image subtraction method (Alard 2000; Alard & Lupton 1998) to the extensive M3 dataset previously analyzed by Corwin & Carney (2001) using DAOPHOT and ALLSTAR. This new analysis has produced light curves and periods for fifteen variables not found in the previous study, but alread known to be variables (see Bakos et al., 2000, catalogue), and has also resulted in improved periods for several other variables. The additional variables recovered with the image subtraction analysis are in the very central region of M3, where crowding is severe and the photometry was not of sufficient quality that it could be put on the standard system. The present study brings to 222 the total number of RR Lyrae variables in Corwin & Carney (2001) M3 dataset, for which light curves and periods are available. Among them we have identified three new candidate double-mode pulsating variables (V13, V200, and V251) reported here for the first time. This brings to 8 the total number of double-mode RR Lyrae (RRd's) identified in M3. Of the newly discovered RRd's V13 is unusual in that it has the fundamental as the dominant pulsation mode. M3 is unique among the globular clusters in having RRd variables with a dominant fundamental mode. Two

[1]Visiting Astronomer, Kitt Peak National Observatory, National Optical Astronomy Observatories, which are operated by AURA, Inc., under cooperative agreement with the National Science Foundation.

of the new candidate RRd's (V13 and V200) have period ratios as low as 0.738-0.739. They lie well separated from all previously known double-mode variable stars in the Petersen diagram, in positions implying a large spread in mass and/or, less likely, in heavy element mass fraction, among the M3 horizontal branch (HB) stars. We explore mass transfer and helium enhancement as possible explanations for the apparent spread in HB masses. We also note that the masses derived from the double-mode analyses now favor little mass loss on the red giant branch.

We find that V200 has changed its dominant pulsation mode from fundamental to first overtone, while V251 has changed its dominant mode from first overtone to fundamental in the interval 1992 to 1993. Together with M3-V166 (Corwin et al. 1999) this is the first time that double-mode variables are observed to switch their dominant pulsation modes while remaining RRd's. The phenomenon is found to occur in a one year time-span thus suggesting that these stars are undergoing a rapid evolutionary phase, and that both redward and blueward evolution may take place among the horizontal branch stars in the Oosterhoff type I cluster M3.

The unusual behavior of the M3 RRd's is discussed in detail and compared to that of the double-mode RR Lyrae identified so far in globular clusters and in the field of our and other Local Group galaxies. We find lack of correlation between the presence of RRd variables and any of the cluster structural parameters.

Key words: globular cluster: individual (M3) — RR Lyrae variables, double-mode pulsators — stars: evolution — stars: fundamental parameters — stars: oscillations — stars: variables: other

1. INTRODUCTION

M3 (NGC 5272) is among the most important and extensively studied Galactic globular clusters. Often considered as a prototype of globular clusters of intermediate-poor metallicity, M3 contains the largest number of RR Lyrae variable stars (N_{RR}) within a single cluster ($N_{RR} \geq 182$, Clement et al. 2001) and is among the ten Galactic clusters with highest specific frequency of such variables (S_{RR}=49.0, from the 2003 update to Harris, 1996, catalogue on Globular clusters, available at http://www.physics.mcmaster.ca/Globular.html).

Since the pioneering study by Sandage (1953), M3 has been the subject of a very large number of photometric surveys. The most recent ones include Buonanno et al. (1994), Ferraro et al. (1997a), and studies based on Hubble Space Telescope (HST) data (Ferraro et al. 1997b,c, and Rood et al. 1999). The color-magnitude diagram (CMD) of M3 displays a horizontal branch (HB) spanning a very wide range in color and a quite narrow red giant branch (RGB; Ferraro et al. 1997a). The color of the RGB is known to depend on metal abundance (see for instance Renzini 1997, and Buonanno, Corsi & Fusi Pecci 1981). Its intrinsic width, after observational effects are removed, is an indicator of metallicity spread and provides upper limits to the dispersion of the elements

having "low" ionization potential (e.g iron, see for instance Suntzeff 1993). Thus the narrowness of the M3 RGB suggests a homogeneous iron abundance of the M3 stars. Suntzeff (1993) reports an upper limit of σ[Fe/H] <0.03 dex to the metallicity dispersion in M3 based on a number of photometric and spectroscopic studies of the cluster.

Recent detailed spectroscopic abundance analyses based on high resolution spectroscopy confirm the very low dispersion in iron abundance of the M3 stars, although some spread exists among independent [Fe/H] estimates in the literature: $[\mathrm{Fe/H}]_{\mathrm{II}} = -1.50\pm0.03$ (Kraft & Ivans 2003), $[\mathrm{Fe/H}] = -1.49\pm0.02$ (from Kraft et al. 1999), -1.34 ± 0.06 (Carretta & Gratton 1997), -1.47 ± 0.03 (Kraft et al. 1992). Spectroscopic observations also reveal α-capture elements enhancement by $\sim$+0.3 dex (Armosky et al. 1994, Carney 1996, Salaris & Cassisi 1996) and star-to-star variations and inhomogeneities in the abundances of the CNO group elements (Suntzeff 1981, Norris & Smith 1984, Kraft et al. 1992, Smith et al. 1996, Lee 1999, Pilachowki & Sneden 2001) among the M3 giants, with both oxygen-rich ([O/Fe]$\simeq$=+0.3) and oxygen-poor ([O/Fe]$\simeq$=$-$0.15) stars coexisting in the cluster (Kraft et al. 1992). However, this is not in contrast with the small spread in color of the RGB since these metals have "high" ionization potentials and their variation is not expected to spread out significantly the RGB [Renzini 1977; Rood 1978 (unpublished)].

M3 contains an extremely rich population of variable stars ($\mathrm{N_{var}}$=274 according to Bakos, Benko & Jurcsik 2000) mainly consisting of RR Lyrae variables, but including also SX Phoenicis stars, and a few long period variables (semi-regular and W Vir stars). The first modern studies of the variable star content of M3 date back to the photographic surveys of Roberts & Sandage (1955), Baker & Baker (1956), and Sandage (1959). More recent studies include the CCD photometric surveys by Kaluzny et al. (1998) and Carretta et al. (1998; 60 variables), and the new catalogue by Bakos et al. (2000) who presented improved identification and astrometry for all known or suspected variables in M3. However, the most extensive study of the M3 variables is that of Corwin & Carney (2001, hereafter CC01) who obtained *BV* CCD photometry, light curves, and ephemerides for 207 of the RR Lyrae variables, and O$-$C diagrams for a subsample of 127 of them. More recently, Strader, Everit & Danford (2002), have presented an image subtraction analysis (Alard 2000, Alard & Lupton 1998) of the variables in the core of M3, adding 11 new candidates (among which 10 possible RR Lyrae stars) to the already overwhelming list of variable stars in M3.

The total number of confirmed and/or suspected RR Lyrae stars identified in M3 by the above studies is larger than 230. Of them about 76 % are fundamental mode pulsators (RRab), 22 % are first overtone pulsators (RRc), and 8 are double-mode variables (RRd), of which 3 were identified in the present study. The transition between fundamental and first overtone pulsators occurs at $\mathrm{P}_{tr} \sim 0.45$ d (see Figures 3 and 7 of CC01), and the average period of the fundamental mode pulsators is $<\mathrm{P}_{ab}>$=0.561 d (CC01), thus making of M3 the best example of Oosterhoff (1939) type I (Oo I) clusters.

In this paper we present a new analysis of the CC01 data using the image-subtraction technique (Alard 2000; Alard & Lupton 1998). The new analysis, described in Section 2, resulted in the

recovering of additional variables not found by CC01, and in improved periods. The new period determination is described in Section 3. Three new candidate double-mode pulsating variables (V13, V200, and V251) were identified with the present study (Section 4). Their pulsational properties are discussed in Section 5 where we also present a detailed comparison with other known cluster and field RRd's.

2. IMAGE SUBRACTION PHOTOMETRY

The photometric data used in the present study were obtained with the 0.9 m telescope at the Kitt Peak National Observatory in three observing runs spanning a period of five years (six nights in 1992, seven nights in 1993, and one night in 1997). They consist of 190 B and 189 V frames with typical exposure times of 500 s and 300 s each, respectively. A complete description of the observations and data processing can be found in CC01. In an attempt to improve the quality of the light curves and to recover known variables (see Bakos et al., 2000, catalogue) that had not been found by CC01, we employed the image subtraction package ISIS V2.1 (Alard & Lupton 1998; Alard 2000) on the extensive dataset of CC01.

The ISIS analysis measures the difference in flux for stars in each image of the time series relative to their flux in a reference image obtained by stacking a suitable subset of images, and after convolving the images with a kernel to account for seeing variations and geometrical distortions of the individual frames. We used the 10 B and the 10 V images with the best seeing of the 1992 and 1993 runs, respectively, to build up the reference images. The new analysis has produced light curves and periods for fifteen variables (V180, V200, V210, V242, V244, V249, V250, V251, V253, V254, V255, V257, V264, V270, V271) that had not been found in CC01 study. The analysis has also resulted in improved periods for other variables. The recovered variables are all located in the very central regions of M3. In an attempt to convert the ISIS differential flux data to standard magnitudes we used DAOPHOT and ALLSTAR to obtain instrumental magnitudes for the new variables in the B and V reference images of the ISIS reductions. Due to crowding in the center, the photometry for the additional 15 variables not found by CC01 was unreliable. All had either high chi values, large magnitude errors or both. Almost all were overluminous for horizontal branch stars probably indicating that they are unresolved blends. No further attempt was made to put these stars on the standard system.

3. NEW PERIOD DETERMINATIONS

Periods for ten of the 15 variables we recovered were determined using the period-search program KIWI, kindly provided to us by Dr. Betty Blanco. KIWI searches for periodicity by seeking to minimize the total length of the line segments that join adjacent observations in phase space, i.e., to maximize the smoothness of the light curve.

Ephemerides for V250, V264, V270 and the three newly discovered double-mode RR Lyraes (V13, V251, and V200) were derived with GRATIS (GRaphycal Analyzer of TIme Series) a private software developed at the Bologna Observatory by P. Montegriffo, G. Clementini and L. Di Fabrizio (see Clementini et al. 2000). GRATIS performs a period search according to two different algorithms : (a) a Lomb periodogram (Lomb 1976, Scargle 1982) and (b) a best-fit of the data with a truncated Fourier series (Barning 1963). GRATIS also allows pre-whitening the data for the first periodicity, and searching for a second periodicity on the residuals with respect to the first periodicity best fit model.

Periods and epochs of maximum light for the fifteen variables not present in CC01 are given in Table 1 along with the new periodicities we derived for M3-V13, an RR Lyrae star reported to have non-repeating light curves by CC01. We also provide in column 4 of the table the periods recently derived for some of these variables by Strader et al. (2002).

The data from our three observing runs cover 1867 days, spanning about 6900 cycles for the shorter period variables, and about 2300 cycles for the longer period ones. Differential B flux light curves for the new variables based on the 1993 data and on the ephemerides given in column 2 and 3 of Table 1 are shown in Figure 1. The different symbols represent data from each of the seven nights of the 1993 run. The double-mode pulsators are plotted using the dominant period. Photometry on a calibrated magnitude scale is available only for V13. In addition to the 15 variables recovered in this study that were not found by CC01, significantly improved light curves were obtained for many variables, some resulting in improved periods. Table 2 lists variables whose new periods differ by more than 0.0001 d from those given in CC01. The variable name in parentheses in Table 2 is the designation of that variable in CC01.

The CC01 analysis and our current study were either unable to obtain data for the following numbered variables listed in Bakos et al. (2000) or found them not to be variables. The parenthetical comments refer to information from Bakos et al. (2000). V164 (possible variable of unknown type); V185 (variability not detected); V198 (variability not detected), CC01 did not observe this star and erroneously misclassified V245 as V198; V217 (RRab); V224 (variability suspected), this variable is present in the CC01 and Strader et al. (2002) data, but both analyses failed to detect variability; V230 (unknown variable type); V233 (variability not detected); V237 (SX Phe); V238 (variability not detected); V262 (not distinguishable from V241 except by HST); V263 (SX Phe); V265 (RR Lyrae blended with V154 and V268); V266 (RRc); V267 (SX Phe); V268 (RR Lyrae blended with V154 and V265); V269 (RRc).

Strader et al. (2002) performed an image subtraction analysis of the variables in the core of M3. Their data consisted of 61 V band images obtained over a five month interval in 2001. They provide a period for one RR Lyrae variable, V262, that was not found in either CC01 or the current study. They also provide a period for the RRab V229 which was found in our current study, but whose light curve was extremely noisy. Their period of 0.6877 phases our data somewhat. Strader et al. were able to find an additional nine variable stars not found in CC01 but that are part of the

present analysis (namely, V200, V242, V250, V253, V254, V257, V264, V270 and V271). They also provided improved periods for several variables listed in CC01. However, many of their periods did not phase our data properly. Strader et al. identified also eleven new suspected variables in M3, among which 10 were classified as RR Lyrae stars. Our data did not include their S1, S2, S5, S8 (a long period variable), and S10. Of the remaining suspected variables, there was too much scatter in our data to produce convincing light curves for any periods. There is some indication in our data that S3 and S7 may be variable.

4. DOUBLE-MODE PULSATORS

Double-mode variables are stars that pulsate simultaneously in both the fundamental and the first overtone radial pulsation modes. First evidence for double-mode pulsation was the discovery that the field RR Lyrae star AQ Leo (Jerzykiewicz & Wenzel 1977, Jerzykiewicz, Schult, & Wenzel 1982) had a secondary first overtone periodicity beside its primary fundamental pulsation period. The first variable of this type in a globular cluster was discovered by Goranskij (1981, V68 in M3). Since these early discoveries an increasing number of double-mode RR Lyrae variables have been identified. Sandage, Katem, & Sandage (1981) found that some variables in the globular cluster M15 exhibit unusually large scatter in their light curves and suggested they might be double-mode pulsators. Cox, Hodson, & Clancy (1983), and Nemec (1985b) confirmed this finding and Nemec designated these stars RRd variables. Additional RRd variables have been found in a number of other globular clusters, among the field Milky Way (MW) RR Lyraes, and in several nearby galaxies.

Double-mode RR Lyrae variables have great importance in constraining certain stellar parameters. It is well known that the pulsation periods depend on basic stellar parameters such as luminosity, effective temperature, mass, and chemical composition, most importantly on the metal content (Kovacs 2001b and references therein). RRd's can be used to estimate the mass and the mass-metallicity relation of HB stars. Masses of double-mode pulsators are evaluated from the ratio between the first overtone (P_1) and the fundamental (P_0) pulsation periods through the so called "Petersen diagram" (Petersen 1973). Pulsation models trace loci of constant mass in this diagram that plots the P_1/P_0 ratio versus P_0, from which stellar masses can be estimated.

The existence of double-mode pulsators suggests the possibility that these stars might be switching pulsation modes from overtone to fundamental or vice versa while evolving across the HB instability strip. Thus they could provide information on the direction and rate of evolution on the HB. Very recently Clementini et al. (2003b) found that the LMC double-mode pulsators are systematically offset to slightly higher luminosities in the V_0 versus [Fe/H] relation defined by the LMC RR Lyraes, thus providing support to the hypothesis that they may be more evolved than the single-mode RR Lyrae stars. Cox et al. (1983) point out, however, that the theoretical switching time is only about 150 years, much too short to account for the large observed number of RRd variables. Nonetheless, there is other evidence that the RRd behavior can change on a short

time scale (Clement et al. 1993, Jurcsik & Barlai 1990). The results presented in the present paper show that this is in fact the case for many of the double-mode RR Lyrae in M3. The physical mechanism for long-term maintenance of double-mode pulsation is still not well understood.

It is interesting to note that RRd variables occur only in a very small percentage of the Galactic globular clusters (GGCs) and appear also to be quite rare in the field of our Galaxy. They are instead more frequent in the field of other Local Group dwarf galaxies, where the new surveys based on large field telescopes and the continuous monitoring of the microlensing studies are constantly increasing their number.

One RRd was identified in both NGC 2419 and NGC 6426 by Clement et al. (1993), 5 in M3 (Corwin, Carney & Allen 1999 and reference therein) and 3 more are reported in the present paper, 12 in M68 (Walker 1994), 14 in M15 (Nemec 1985b), and 17 in IC 4499 (Walker and Nemec 1986). Many other clusters have been searched with only negative results. Even the very RR Lyrae-rich cluster ω Cen has no RRd variables (Nemec et al. 1986). Only 6 RRd's were known so far in the field of our Milky Way (see Clementini et al. 2000 and references therein), and the search in OGLE-II extensive database for variable stars in the Galactic bulge has added only 3 further candidates (Mizerski 2003). However, Cseresnjes (2001) found 13 more Galactic RRd variables lying along the line of sight to the Sagittarius dwarf galaxy.

Turning to the extragalactic Local Group systems: 6 RRd candidates are reported in Carina by Dall'Ora et al. (2003); 10 RRd's have been identified in Draco (Nemec 1985a); 12 in the field of Fornax and 8 in the globular cluster Fornax 3 (Clementini et al. 2003a) [2]; 18 in Sculptor (Kovacs 2001a); 40 in Sagittarius (Cseresnjes 2001); and 57 in the Small Magellanic Cloud (Soszyński et al. 2002). 181 RRd's were found in the field of the Large Magellanic Cloud by Alcock et al. (1997, 2000) on the basis of MACHO microlensing observations. This sample has been extended to 230 by Soszyński et al. (2003) using OGLE-II database. The same authors also identified 6 and 2 RRd's respectively in the LMC globular clusters NGC1835 and 2019, while 4 RRd's are reported in Reticulum by Ripepi et al. (2003).

In Tables 3, 4 and 5 we have collected information on the RRd variables identified so far in the various stellar systems.

Table 3 summarizes the literature data for the globular cluster RRd's. In this table we provide: total number of RR Lyrae stars in the cluster: $\mathrm{N_{RR}}$; number of double-mode RR Lyrae: $\mathrm{N_{RRd}}$; cluster metallicity: [Fe/H] on the Zinn & West (1984, hereafter ZW84) scale; classification of the clusters in Oosterhoff types (Oosterhoff 1939); specific frequency of the RR Lyrae variables: $\mathrm{S_{RR}}$= $\mathrm{N_{RR}} \times 10^{0.4(7.5+M_\mathrm{V})}$, which is the number of RR Lyrae stars in a cluster ($\mathrm{N_{RR}}$) normalized to a cluster absolute magnitude of $M_V = -7.5$; horizontal-branch ratio: HBR=(B−R)/(B+V+R) where B and R are the numbers of HB stars respectively to the blue and to the red of the RR Lyrae

[2]The number of field RRd's in Fornax is likely a lower limit since these authors discuss only the variables they detected in 1/8 of the area of the galaxy they surveyed.

instability strip, and V is the number of RR Lyrae stars; and cluster structural parameters: central concentration: c=$log(\mathrm{r_t}/\mathrm{r_c})$; core radius: $\mathrm{r_c}$; half-mass radius: $\mathrm{r_h}$, and tidal radius: $\mathrm{r_t}$) of the host clusters. Proper references for each of these quantities are given in the notes to Table 3. Bragaglia et al. (2001, hereafter B01) have estimated "pulsational" masses for the Galactic cluster RRd's and for some of the field RRd's in the MW and in the LMC, using the Petersen diagram and an extension of Bono et al. (1996) non-linear convective pulsational models. Columns 12 and 13 of Table 3 provide the average mass of the MW cluster RRd's and its dispersion. These averages are the simple mean of the "pulsational" masses derived for individual stars by B01. Notice that masses being derived for RR Lyrae stars from the new pulsation models are now consistent with very little mass loss on the red giant branch. That is, they are indistinguishable from those of the main sequence turn-off. Moreover, as already noticed by B01, the new models no longer predict a difference in mass between Oosterhoff type I and II clusters, or, depending on the adopted metallicity scale (ZW84 or Carretta & Gratton 1997), the difference is opposite to what was previously thought (see B01 for details).

Table 4 summarizes information on the field RRd's identified so far in the various Local Group galaxies. Metal abundances in Table 4 refer to the whole systems and not to individual RRd's. They are on the ZW84 scale, and generally refer to the abundance of the old stellar component in these galaxies, which in several cases (e.g. Fornax, Sagittarius, etc.) is derived from the pulsational properties of the fundamental mode RR Lyrae stars. For the LMC we list in Table 4 the average metallicity recently derived from the spectroscopic study of 100 LMC RR Lyrae stars by Clementini et al. (2003b), and Gratton et al. (2003, in preparation). This value has been transformed to the ZW84 metallicity scale according to Clementini et al. (2003b). The Oosterhoff type classification of the Local Group galaxies has been assigned from the average period of the field ab type pulsators in each galaxy, according to: Alcock et al. (1996) for the LMC, Soszynski et al. (2002) for the SMC, Cseresnjes (2001) for Sagittarius, Kaluzny et al. (1995) for Sculptor, Clementini et al. (2003a) for Fornax, Nemec (1985a) for Draco, and Dall'Ora et al. (2003) for Carina. For the Milky Way it refers to variables in the Galactic Center (Cseresnjes 2001).

Individual metal abundances are available only for 4 of the 6 Milky Way field RRd's and for 9 of the 230 LMC RRd's, and these are provided in Table 5. These metallicities are either on the ZW84 scale, or on Clementini et al. (1995) scale. These two scales are quite similar to each other (B01).

At the moment there is no clear observational indication that the occurrence of the RRd phenomenon can be related to any specific cluster or stellar property (Kovacs 2001b). The RR Lyrae phenomenon is itself related to metallicity since RR Lyrae stars are rarely found in metal-rich clusters, and the frequency and mean periods depend on metallicity among the metal-poor clusters. The metallicity distribution of all known RRd's is shown in Figure 2. In general only metal poor systems ([Fe/H]< -1.5) seem to contain RRd variables, with only three of the field RRd's in the LMC with metallicity extending to values larger than this limit. The open triangle with the arrow in Figure 2 plots the total number of the LMC field RRd's assuming the average metal abundance of the LMC RR Lyrae in Clementini et al. (2003b), transformed to the ZW84

scale.

As for the globular clusters, only a few clusters more metal rich than [Fe/H]=-1.0 contain RR Lyrae stars. The 6 Galactic clusters containing RRd's all are more metal-poor than [Fe/H]=-1.5, and 4 of them are at the very metal-poor edge of the Galactic globular cluster metallicity distribution ([Fe/H]≤ -2.0). Similarly, the 4 extragalactic globular clusters containing RRd's all are more metal-poor than [Fe/H]=-1.5. This is shown in Figure 3, where we plot the specific frequency of the RR Lyrae variables: $\mathrm{S_{RR}}$, and the horizontal-branch ratio: HBR, as a function of the cluster metal abundance. Filled symbols mark the clusters where double-mode RR Lyrae stars have been detected. Some further parameter other than metallicity must determine why only these 10 metal-poor clusters contain RRd's. The RR Lyrae specific frequency may be relevant, since 5 out of the 10 clusters hosting double-mode variables have $\mathrm{S_{RR}}$ values larger than 40, and two of them (Reticulum and IC4490) are among the clusters with highest RR Lyrae specific frequency in general. Seeking for correlations between number of double-mode variables and properties of the host cluster, in Figures 4 and 5 we plot the number of double-mode variables ($\mathrm{N_{RRd}}$) versus structural parameters of the 10 globular clusters containing RRd's. When transforming to parsecs the angular values of r_c, r_h, and r_t we have assumed for the Galactic clusters the distances to the Sun provided in Harris catalogue. For the extragalactic clusters we have assumed distances of 51, and 138 Kpc for the LMC, and Fornax dwarf spheroidal galaxy, respectively. In the figures open and filled circles are used for the Milky Way Oosterhoff types I and II clusters (Oosterhoff 1939), respectively; filled triangles represent the LMC clusters, and the filled pentagon is Fornax 3. Among the extragalactic clusters containing RRd's Reticulum is of Oosterhoff type I (Walker 1992), NGC 1835 is an Oosterhoff intermediate (I/II, Soszynski et al. 2003), Fornax 3 is an Oosterhoff type II cluster (Clementini et al. 2003a), while no classification is available yet for NGC 2019. M15 is a core-collapsed cluster and is labelled with a double c in the Figures. No clear correlations are seen in either of these diagrams.

4.1. NEW DOUBLE-MODE RR LYRAE STARS IN M3

M3 was previously known to have 5 RRd variables: V68, V79, V87, V99, and V166 (Goranskij 1981, Cox et al. 1983, Nemec & Clement 1989, Clement et al. 1997, and Corwin et al. 1999). Here we report the discovery of 3 additional candidate RRd variables, V13, V200 and V251.

V13: This variable was discussed in CC01. As the article was going to press it was realized that V13 is probably a double-mode pulsator. A more detailed analysis using both the original data and the more recent image subtraction data has determined that the star has in fact two periodicities with a period ratio close to the canonical value for double-mode RR Lyrae stars, the period ratio range of all the presently known RRd stars being in the range $0.741 < \mathrm{P_1/P_0} < 0.748$ (Popielski, Dziembowski, & Cassisi 2000).

Figure 6 shows the periodograms for the DAOPHOT V data of V13 obtained with GRATIS using the Lomb algorithm to identify the most probable frequency of the data on a wide interval

of 0.22-0.8 d. Exactly the same periodicities are found in the full 1992 + 1993 +1997 data-set (top panel of Figure 6), and when the 1992 (middle panel), and the 1993 + 1997 (lower panel) data subsets are analyzed separately. These correspond to a dominant period of about 0.476 d (fundamental mode, frequency of 2.10 /d) and a secondary period of about 0.351 d (first overtone mode, frequency 2.85 /d). The frequency 3.10 /d is an alias of the fundamental mode and the frequency 1.85 /d is an alias of the first overtone mode. A more detailed determination of dominant and secondary frequency components was obtained by (i) reducing the interval around the primary periodicity and using GRATIS Fourier algorithm to find the best fit, and (ii) by pre-whitening the data with the dominant primary frequency and searching for the secondary frequency component in the residuals with respect to the best fit model of the primary frequency. The derived periods are P_0=0.47951 ± 0.00001 d and P_1=0.35382 ± 0.00001 d for a period ratio of 0.738. This value is lower than found so far for RRd variables in M3 and for RRd variables in general.

In Figure 7, the top panel shows the light curve of V13 using the DAOPHOT *V* data with no pre-whitening and folded according to the highest amplitude, primary (fundamental) period of pulsation P_0=0.47951 d. The rms deviation from a 6 harmonics best fit model is 0.091 mag (much larger than expected from observational errors alone: 0.02 mag in *V*), the amplitude of the light variation is 0.576 mag, and the intensity-averaged *V* magnitude is 15.649 mag. The middle panel of Figure 7 shows the light curve of the primary (fundamental) period of V13 after pre-whitening of the secondary (first overtone) period P_1=0.35382 d. The amplitude of the light variation is A_0 = 0.534 mag and the average *V* magnitude is 15.662 mag. The residuals from the 6 harmonic best fit model have been reduced to 0.046 mag. Finally, the bottom panel of Figure 7 shows the light curve of the secondary (first overtone) period after pre-whitening of the primary (fundamental) period. The amplitude of the light curve of the secondary period is A_1 = 0.246 mag. Figure 8 shows the light curves of V13 using the DAOPHOT *B* data. The light curve with no pre-whitening (top panel of Figure 8) has residuals from the 6 harmonics best fit model of 0.115 mag (to compare with 0.02 mag of the *B* photometric errors) and residuals of 0.051 after pre-whitening (middle panel). A_0 = 0.693 mag and A_1 = 0.308 mag. The intensity-averaged *B* magnitude, after pre-whitening is 15.930 mag.

A similar analysis of the Alard image subtraction data confirms the double-mode behavior of V13 and the periods given above for the dominant and secondary pulsations.

As is usual in RRd stars the amplitude of the secondary mode is about half that of the dominant mode: A_1/A_0=0.46 and 0.44 in *V* and *B* respectively. However, contrary to the vast majority of the known RRd's, but like M3-V68 in the time interval 1920-1926 (Nemec & Clement 1989), in this star the primary periodicity is the fundamental mode. Indeed, only a very small percentage of the known RRd variables are found to have a dominant fundamental mode: AQ Leo in the Milky Way (Jerzykiewicz & Wenzel 1977, Jerzykiewicz, Schult, & Wenzel 1982), and a few field RRd's in the LMC (Alcock et al. 2000). However, so far M3 is the only cluster where this phenomenon has occurred.

V200: This variable was not found in the original CC01 analysis. The photometry for V200 in the current analysis is not good enough to reliably determine standard magnitudes and the analysis of this variable was based on the Alard B differential flux data. This caused some uncertainty in the period search of the star. Figure 9 shows the periodograms for the 1992 + 1993 + 1997 data (top panel), the 1992 data (middle panel), and the 1993 + 1997 data (bottom panel) obtained with GRATIS searching the data with the Lomb algorithm on a wide interval of 0.22-0.8 d. The top panel of Figure 9 shows that there are two frequencies of about equal strength, one of about 0.488 d (fundamental mode, frequency of 2.05 /d) and another of about 0.357 d (first overtone mode, frequency 2.80 /d). The frequencies 3.05 and 4.05 /d are aliases of the fundamental period and the frequencies 1.80 and 3.80 /d are aliases of the first overtone period. A further search using the Fourier algorithm and pre-whitening the data with the dominant primary periodicity gives most likely periods of 0.4867 d and 0.3598 d, for a period ratio of 0.739. This value is lower than expected for RRd variables in M3 and for RRd variables in general, but slightly higher than V13 discussed above. It should be noted, however, that period search solutions providing fundamental and first overtone periodicities with period ratios as short as 0.735-0.734 d are also compatible with the differential flux data of V200. However, the first solution is preferred since it provides a period ratio closer to the canonical values observed for RRd stars.

A most important feature in Figure 9 is that when the 1992 and 1993+1997 data are analyzed separately it is found that in 1992 the fundamental and first overtone frequencies are of about equal strength (see middle panel of Figure 9). However, in the 1993 + 1997 data, the first overtone frequency is clearly dominant (see bottom panel of Figure 9). This seems to suggest that V200 may have switched its dominant pulsation mode from fundamental to first overtone. A very similar behavior was observed in M3-V166 by Corwin et al. (1999) who employed a different period search algorithm than here, to study the star, the phase dispersion minimization technique (PDM) developed by Stellingwerf (1978).

The top panel of Figure 10 shows the light curve of V200 using the full 1992 + 1993 + 1997 Alard differential flux data with no pre-whitening and folded according to the highest amplitude, primary (first overtone) period of pulsation P_1=0.3598 d. The middle panel shows the light curve of the primary (first overtone) period of V200 after pre-whitening of the secondary (fundamental) period P_0=0.4867 d. Finally, the bottom panel shows the light curve of the secondary (fundamental) period after pre-whitening of the primary (first overtone) period.

V251: As was the case for V200, this variable was not found in the original CC01 analysis and the photometry in the current analysis is not good enough to reliably determine standard magnitudes for V251. Thus, also for this star the period search was based on the Alard B differential flux data. However, contrary to V200, for V251 it was possible to derive definite periodicities. Figure 11 shows the periodograms for the 1992 + 1993 + 1997 data (top panel), the 1992 data (middle panel), and the 1993+1997 data (bottom panel) of the star obtained by GRATIS using the Lomb algorithm. The top panel shows a dominant period of about 0.474 d (fundamental mode, frequency of 2.11 /d) and a secondary period of about 0.353 d (first overtone mode, frequency 2.81

/d). The frequency 3.11 /d is an alias of the fundamental mode and the frequencies 3.81 and 1.81 /d are aliases of the first overtone mode.

The dominant periodicity, best fitting the data with a 5 harmonics Fourier series, is P_0=0.47423 ± 0.00001 d. Data pre-whitened according to the above primary period are then best fitted by a two harmonics Fourier series corresponding to a secondary periodicity of P_1=0.35384 ± 0.00001 d. The corresponding period ratio of 0.7461 is in the range typical of RRd variables in M3.

When the 1992 and 1993+1997 data are analyzed separately it is found that in 1992 the dominant mode was the first overtone for P_1=0.35384 d, while in 1993+1997 the dominant mode is the fundamental one P_0=0.47423 d (see Figure 11 middle and bottom panels). Like V13, the present primary periodicity of V251 is the fundamental mode, however the behavior of the periodograms suggests that V251 may have switched its dominant pulsation mode from first overtone to fundamental in the interval 1992 to 1993.

The top panel of Figure 12 shows the light curve of V251 using the full 1992 + 1993 + 1997 Alard differential flux data with no pre-whitening, and folded according to the highest amplitude, primary (fundamental) period of pulsation P_0=0.47423 d. The middle panel shows the light curve of the primary (fundamental) period of V251 after pre-whitening of the secondary (first overtone) period P_1=0.35384 d, and finally, the bottom panel shows the light curve of the secondary (first overtone) period after pre-whitening of the primary (fundamental) period.

Fundamental periods (P_0) and period ratios (P_1/P_0) for the 8 RRd's in M3 are summarized in Table 6 along with intensity-averaged magnitudes and colors (columns 5 and 6) taken from CC01, and distances from the cluster center (column 7). In column 4 of the Table we also list pulsational masses derived by B01 for the already known M3 RRd's, and rough estimates of the mass of the newly discovered RRd's, derived as described in Section 5.1.

5. DISCUSSION

M3 seems to be somewhat unusual with respect to its double-mode pulsators. We summarize here the distinctive characteristics of the cluster's RRd variables. The cluster is unique among the globular clusters in that it has RRd variables with a dominant fundamental mode: V13, V166 (in 1992, but not 1993), V251 (in 1993, but not 1992), and possibly V68 (see discussion in Corwin et al. 1999). Of them V251 is located in the cluster core, while the remaining three do not have a preferred physical location in the cluster, or occupy any different part of the CMD. Only very few RRd variables are presently found to have a dominant fundamental mode, and they all are field variables (AQ Leo, and a few RRd's in the LMC, Alcock et al. 2000).

M3 is so far the only cluster in which RRd variables have been observed to switch from one dominant mode to another while remaining RRd variables (Corwin et al. 1999 and the present analysis). Variations in the double-mode behavior have been observed in a number of RRd vari-

ables in M68 (V33 and V21, Clement et al. 1993), M15 (V30 and V53, Purdue et al. 1995, V26, Jurcsik & Barlai 1990), M3 itself (V79, Clement et al. 1997), and in the Galactic field RRd AQ Leo (Jerzykiewicz, Schult, & Wenzel 1982). Most of these variations concern (i) initiation or cessation of the double-mode behavior, and (ii) changes in the length and amplitude ratios of the two periodicities. However, none of these studies reports a switch between dominant modes to have actually occurred in these stars. We will come back to this point in Section 5.2.

M3 also has an unusually large range of period ratios, not only for a single cluster but for RRd variables in general. It was generally assumed that RR Lyrae stars which pulsate simultaneously in the fundamental and first overtone modes have period ratios P_1/P_0 between 0.73 and 0.76 d (Cox et al. 1983, Nemec 1985b). Kovacs (2001a,b) reports that all known RRd stars have period ratios in the range of 0.741 and 0.748, and that variables within a single cluster cover only narrow ranges within this interval. In general, metal-poor clusters have RRd stars with larger period ratios and periods, while for metal-rich clusters these values are smaller. M3 does not seem to conform to this rule. Six of the eight RRd variables have period ratios in the range 0.743 and 0.747 (Corwin et al. 1999 and the current analysis) while the other two, V13 and V200 have period ratios of 0.738 and 0.739 respectively.

An important question is whether, with proper observations, anomalies similar to those seen in some of the M3 RRd's could be found also in other cluster and field RRd's. In this respect we note that assets of the present analysis were the relatively extensive and widespread time coverage (6, 7 and 1 consecutive nights, arranged in a 6 years time span), the rather large number (190 B and 189 V) and good photometric precision (0.02 mag in both B and V passbands) of our CCD data for the M3 variables, and the use of a powerful detection technique (e.g. the image subtraction method, Alard 2000, Alard & Lupton 1998). Thus it may be possible that with further observations and the use of appropriate detection techniques allowing to search the crowded cluster cores, new RRd variables and perhaps some of the "anomalies" found among the M3 RRd's could also be detected in other GCs. For instance, it is interesting that the very extensive photometric survey of variable stars resulting from the MACHO project (Alcock et al. 1996) succeeded in identifying some RRd's with dominant fundamental pulsation mode (Alcock et al. 2000, and references therein) in the LMC.

5.1. THE PETERSEN DIAGRAM

Figure 13 shows the position of the M3 RRd's in the Petersen diagram of all known double-mode RR Lyrae stars identified so far in various different stellar systems, for which periods and period ratios are found in the literature. Filled circles are used for the 3 new candidate double-mode stars in M3 while filled squares mark the 5 previously known RRd's. Lines show the pulsational models by Bono et al. (1996), which correspond to Z=0.0001, and $M/M_{\odot}$=0.65, 0.75, and 0.80, and their extension to larger mass and metallicity ranges ($M/M_{\odot}$=0.85, and Z=0.0002, 0.0004, 0.0006, 0.0008) obtained by B01. The derivation of these new models is fully discussed in B01, to

which the interested reader is referred for further details. Figure 13 provides a summary of our present knowledge of both "observational" (periods and period ratios) and "theoretical" (pulsational models) Petersen diagram.

Data plotted in Figure 13, which is an updated version of B01 Figures 5 and 6, were taken from Garcia-Melendo & Clement (1997) for NSV09295; from Clement et al. (1991, 1993) for AQ Leo, VIII-10, and VIII-58; from Clementini et al. (2000) for CU Com; from Moskalik & Poretti (2003) for BW7 V30; from Cseresnjes (2001) for the field MW RRd's in the direction of Sagittarius (13 RRd's); from Clement et al. (1993) for NGC 2419 (1 object), and NGC 6426 (1 object); from Corwin, Carney & Allen (1999) for M 3 (5 objects); from Walker & Nemec (1996) for IC 4499 (17 RRd's); from Walker (1994) for M 68 (12 RRd's); from Nemec (1985b) for M 15 (14 RRd's); from Nemec (1985a) for Draco (10 RRd's); from Kovacs (2001a) for Sculptor (18 objects); from Cseresnjes (2001) for Sagittarius (40 RRd's); from Soszyński et al . (2002) for the SMC (57 RRd's); from Soszyński et al . (2003) for the LMC field RRd's (230 objects) and for the RRd's in the LMC clusters NGC 1835 (6 objects) and NGC 2019 (2 RRd's); from Ripepi et al. (2003) for the Reticulum cluster (4 RRd's). The 6 RRd candidates in Carina, and the field and cluster RRd's in Fornax (12 and 8 objects, respectively), are not shown in the figure since periods are not available yet for these stars.

The large spread in period ratio of the M3 double-mode pulsators, a spread much larger than found in any other of the stellar systems containing RRd's, can clearly be seen in the Figure. Figure 14 shows an enlargement of Figure 13 in the region relevant for the M3 RRd's. Only the M3 double-mode variables are displayed now, and lines of pulsational models corresponding to various values of the heavy element mass fraction Z are labelled by mass and luminosity level, in solar units. Since both mass and metal abundance affect the predicted Petersen diagram, the dispersion of the M3 double-mode variables in Figure 14 can be explained either by a spread in heavy element mass fraction Z, or by a spread in mass.

B01 made the following assumption: (i) [Fe/H]=−1.66 (on the ZW84 scale) and $Z_{\odot}$=0.019, (ii) no significant spread in [Fe/H], as reasonable for stars within a cluster and as confirmed by both spectroscopic determinations[3] and the very low intrinsic width of the M3 Red Giant Branch (see Introduction), and (iii) no α-enhancement. They derived masses with dispersion of about 0.06 $M_{\odot}$ around the average value M=0.779 $M_{\odot}$, for the 5 previously known RRd's in M3. This dispersion is 3 times larger than they find for the double-mode variables in IC4499 and M68, and 1.5 larger than in M15 (see column 13 of Table 3).

With a period ratio of P_1/P_0=0.7461 and a fundamental mode period P_0=0.47423, in the Petersen diagram V251 falls very close to 4 of the M3 RRd's analyzed by B01, namely: V99 (P_1/P_0=0.7468, $M/M_{\odot}$=0.846); V166 (P_1/P_0=0.7459, $M/M_{\odot}$=0.803); V87 (P_1/P_0=0.7458, $M/M_{\odot}$=0.795);

[3]Spectroscopic abundance analyses find a small dispersion in the [Fe/H] abundance of M3, independently of the adopted value and metallicity scale. For instance, Ivans & Kraft (2003) find σ[Fe/H]$\leq$0.03 dex, based on the analysis of 17 M3 red giants.

and V79 (P_1/P_0=0.7453, $M/M_\odot$=0.772). Thus according to B01 analysis V251 could have a mass around $\sim$0.8 $M_\odot$.

With their significantly lower period ratios, V13 (P_1/P_0=0.738) and V200 (P_1/P_0=0.739) lie well separated from all the known RRd's and in-between the Z=0.0008, $M/M_\odot$=0.65, log $L/L_\odot$=1.61, 1.72 model lines. Thus the comparison with the nonlinear pulsational models seems to suggest that V200 could be a $\sim$ 0.65 $M_\odot$ pulsator with a luminosity level intermediate between 1.61 and 1.72 and a total heavy-element mass fraction Z$\sim$0.0008. V13 could be either a lower mass variable (M $\sim$ 0.55 $M_\odot$) with the same metal abundance of V200, or a more metallic pulsator (Z$\sim$0.001) with mass similar to V200[4]. The higher global abundances of V200 and V13 should be the result of a non negligible and varying α-enhancement (α=0.2−0.4 dex for $Z_\odot$=0.02) in these two variable giant stars. A varying α-enhancement and variations in oxygen abundance in particular would not be in contrast to the absence of star-by-star scatter in [Fe/H], since, as discussed by VandenBerg (1992) and Rood & Croker (1985), variations in oxygen do not affect the radii, hence the colors of the low-mass, low-metallicity red giants. Such variations are expected, however, to affect the effective temperature hence color of stars near the main-sequence turnoff, though not to an easily observable extent. However, observations show that the M3 giants have an almost constant α-enhancement (O, Si, Ca, Ti) by about 0.3 dex (Armoski et al. 1984, Carney 1996, Salaris and Cassisi 1996, and references therein).

One may wonder whether the star-to-star variations in the elements of the CNO group reported in M3 by several independent spectroscopic studies may produce a heavy element mass fraction variation among the M3 horizontal branch (HB) stars. Variations in both C and N abundances and carbon-nitrogen anticorrelation, with a constant total abundance of C+N, were first discovered among the M3 giants by Suntzeff (1981), and later confirmed by Norris & Smith (1984) and Lee (1999). Oxygen variations with both oxygen-rich [O/Fe]$\simeq$=+0.3 and oxygen-poor [O/Fe]$\simeq$=−0.15 stars coexisting in the cluster, and oxygen-sodium anticorrelation (with mainly O rich and Na poor stars among the M3 giants), were found by Kraft et al. (1992). Finally, Smith et al. (1996) find anticorrelation between CN band strength and both [O/Fe] and [C/Fe] abundances, coupled with CN-[N/Fe] correlation, but they also show that the total [(C+N+O)/Fe] is constant among the M3 giants.

Thus the chemical mix is changing, but the total heavy element mass fraction, Z, is probably not changing, making very unlikely to explain the position of V200 and V13 in the Petersen diagram in terms of a difference in Z. Clearly, high resolution spectroscopy and direct measure of the elemental abundances of the M3 double-mode pulsators are highly desirable in order to break the mass-metallicity degeneration in the Petersen diagram, due to the similar effects of the metallicity and mass on the period ratio.

---

[4]If the above mass evaluations are correct, the average mass spread of the M3 RRd's would be as large as $\sim$0.10 $M_\odot$.

Similarly, if some sort of mass transfer is likely to cause these low masses, then radial velocity monitoring should be able to reveal possible binary systems. Gunn & Griffin (1979) and Pryor, Latham & Hazen (1988) have searched for binaries among the red giants in M3. Neither study found a high binary fraction. However, a re-analysis of the data from Pryor et al. is in progress. It may be daunting to reveal binarity in the presence of pulsation, especially multimode pulsation, but periodograms ought to pull out a period well enough, and if photometry is used to monitor the changes in temperature and radius, one might still be able to obtain an orbital solution. The orbital period is likely to be much longer than the pulsational periods and hence be extractable via Fourier analysis technique[5].

Also further photometric observations of the M3 RRd's with improved spatial resolution and covering larger time intervals are needed to remove the uncertainties still present in the period determinations (particularly for M3-V200) and to monitor period/mode changes in these stars.

On the other hand, the very wide range in color (temperature) of M3 horizontal branch shows that some spread in mass exists among the M3 HB stars. CC01 find also that the HB of M3 has a large vertical height of the order of about 0.3 mag with a number of variables both subluminous and overluminous than the zero age horizontal branch (ZAHB), that they set at V=15.72 mag. In fact, 12 RR Lyrae stars in CC01 sample lie below the ZAHB at an intensity-averaged V magnitude around $V\sim$15.826 mag, and 30 variables lie above the ZAHB at an intensity-averaged V magnitude brighter than $V\sim$15.55 mag. CC01 interpret these differences as possibly produced by both mass dispersion and evolutionary effects occurring among the M3 RR Lyrae stars.

The subluminous RR Lyrae stars are located within 81 arcsec from the cluster center, thus CC01 discuss whether a lower HB core mass resulting from star-star interactions in the high density cluster central region might be responsible for the lower luminosity of these stars. Our results on the M3 double-mode pulsators, with two of the newly discovered RRd's being significantly less massive than the other M3 RRd's, seem to confirm that an unusually large spread in mass is likely to exist within the M3 HB stars, and in particular in the very narrow region of the HB instability strip corresponding to the transition between fundamental and first overtone pulsators. M3 has also many blue stragglers, especially toward the center (Ferraro et al. 1997c) where binaries are likely to be more common. Thus one possibility is whether the subluminous RR Lyrae and the small mass RRd variables could have acquired atypical masses from some phenomenon related to the core properties of the cluster. We have no radial velocity information but there are suggestions that mass-transfer binaries may be more common in clusters' cores (Hut et al. 1992).

Among the two new candidate RRd's with possibly unusual small masses only V13 has calibrated photometry so we can check its position on the CMD. According to the intensity-averaged magnitudes listed in Table 6 the RRd variables with calibrated photometry have av-

[5]The RR Lyrae stars are descendents of red giants whose radii may have been as large as 100 $R_{\odot}$, and stellar companions capable of accepting mass from the red giant (pre-RR Lyrae star) would likely have had orbital periods of weeks to months to, perhaps, years.

erage luminosity $< V_{RRd} >$=15.63±0.04 and lie close and slightly above the ZAHB level (δV_{RRd}= $< V_{RRd} > - < V_{ZAHB} >$=$-0.09 \pm 0.04$ mag. In particular, V13 ($< V_{int} >$=15.612 and 15.662 mag in CC01 and in the present analysis, respectively) lies about 0.1 mag above the ZAHB. Thus if the star is actually less massive, evolution (or some other mechanism/parameter) must have compensated in part the lower luminosity corresponding to its lower mass. We will come back to this point at the end of this section.

5.1.1. MASS-TRANSFER IN BINARY SYSTEMS

If according to the Petersen diagram in Figures 13 and 14, the atypical RRd's in M3 are those with unusually small period ratios, that fall in regions where no other RRd has ever been found before (namely V200, V13 and to a lesser extent V68), then according to the pulsational models these stars should have "smaller" masses. We are assuming here that the Petersen diagram is applicable and that it is actually able to predict correct masses for stars undergoing the rapid evolution causing the period changes and the mode switching observed in the M3 RRd's. A further caution is that we may also need improved periods and period ratios for some of our stars, like V200 and V251.

In any case, if mass-transfer in a binary system triggered by the high central density of M3 core is the mechanism causing the spread in mass, these atypical RRd's should be the "donors" of mass of such systems. However, not all of the unusual M3 stars are in the core. Neither are all the blue stragglers, either. In fact, of the 3 RRd's with atypically short period ratios, hence atypically small masses (V13, V200, and V68), only V200 is in the central regions of the cluster (see column 7 of Table 6). V251 is in the core region but it has a period ratio, hence mass, similar to the other "typical" RRd's in M3, while V13, "atypical" period ratio and mass it is not in the cluster core.

Of the GGCs containing double-mode variables M68 also has a widely spread HB with a well extended blue tail and contains several likely blue stragglers (Walker 1994). However, its RRd stars show a very modest mass dispersion ($\sigma_{\rm M} = 0.019 {\rm M}_\odot$); see column 13 of Table 3). Conversely, double-mode variables in M15, a core-collapsed cluster with only a few red HB stars, many variables, and a bifurcated blue HB with a long tail of very blue stars, exhibit mass dispersion ($\sigma_{\rm M} = 0.036 {\rm M}_\odot$) intermediate between M68 and M3. Why do we see a spread in mass in M15 and M3 (clusters with very different HB morphologies) and do not see it in M68 whose HB morphology is intermediate between M3 and M15?

5.1.2. HELIUM ENHANCEMENT

An alternative possible explanation for the spread in mass and luminosity of the M3 RR Lyrae stars could be the presence of a spread in helium abundance within the framework recently suggested by D'Antona et al. (2002). These authors present stellar evolution models for globular cluster

stars with helium enhancement arising from self-pollution of the intracluster material and of the pre-existing main sequence structures by the ejecta of first generations of massive asymptotic giant branch (AGB) stars. As first suggested by Norris et al. (1981) the self-pollution mechanism due to intermediate mass stars would produce the CNO abundance spread and chemical peculiarities observed in cluster stars, but as recognized by D'Antona et al. it would also cause a *helium enrichment*. Isochrones for GC ages computed by D'Antona et al. (2002) show that the most compelling effect of the helium enhancement would be to lower the masses (by up to about 0.08 $M_{\odot}$ for a helium enhancement Y from 0.23-0.24 up to 0.28) of the evolving red giant branch (RGB) stars at the RGB tip, as a result of both evolution and mass loss occuring in the RGB phase. Thus the effect of a distribution in the initial value of the helium abundance would best be seen on the HB where helium-enhanced HB and RR Lyrae stars would, on average, be more luminous and bluer, with a substantial thickening of the HB, than stars with standard low helium abundance. The helium enhancement should not affect, however, the periods of the RR Lyrae stars, since, as shown by Caputo, Santolamazza & Marconi (1998), for fixed values of the structural parameters (mass, luminosity and effective temperatures) the periods of fundamental and first overtone pulsators do not vary for a variation of the helium content in the range $0.24 \leq Y \leq 0.31$ (see Figure 2b of Caputo et al. paper).

The Y enhancement would also produce variations of the luminosity and color of the Main Sequence (MS), Turn-Off (TO) and RGB. D'Antona et al. (2002) estimate that at Z=0.0002 and for a helium enrichment from 0.24 to 0.28 the TO magnitude of the helium enriched stars would be 0.07 fainter, the main sequence location would be 0.13 mag fainter, and both the color along the RGB and below the MS TO would be 0.015 mag bluer. We note that the spread in luminosity and color observed in the subgiant region of M3 (see Figure 4 of CC01, and Figure 15 of Ferraro et al. 1997a) could thus be a signature of helium dispersion among the M3 stars, although, as discussed by Ferraro et al. (1997a), it could also be the result of *optical blending* between MS-stars and between subgiant branch objects and blue stragglers.

Depending on the actual distribution of the helium enriched stars and on the total amount of assumed average mass loss, the mechanism proposed by D'Antona et al. (2002) would explain the various different HB morphologies, including the HB gaps and tails observed in GCs, for values of the parameters largely in the observed range. Of particular interest is the simulation of M3-like HB morphologies (see top right panel of Fig. 4 in D'Antona et al. paper), where both helium-enriched and low-helium stars enter the RR Lyrae strip and well reproduce a large fraction of the vertical height observed in the M3 HB. We specifically note that the predicted total mass spread of 0.08 $M_{\odot}$ (for a helium enrichment up to 0.04) would be consistent with both the spread in mass ($\sigma_{\rm M}$=0.06−0.10 $M_{\odot}$) and the average overluminosity ($\delta V_{RRd}=-0.09$ mag) observed in the M3 RRd variables.

On the other hand, since in D'Antona et al. (2002) scenario the helium enrichment is induced by an intracluster mechanism (self-pollution), whose amount and extent may differ from cluster to cluster depending on "local" conditions as for instance the cluster metal abundance, total mass

and concentration, with a proper fine tuning of the involved parameters it should be conceivable to explain, within the same framework, both the different HB morphology and the smaller spread in mass of the M15 RRd's, and the lack of mass spread observed among the M68 double-mode stars.

5.2. PERIOD CHANGES AND MODE SWITCHING

5.2.1. INTRODUCTION

Changes in the pulsation period are a very common phenomenon among the RR Lyrae stars, and there is growing observational evidence that they occur also among the double-mode RR Lyrae variables. Our results on the new double-mode RR Lyrae stars in M3 confirm this occurrence.

Since RR Lyrae star periods are sensitive to the RR Lyrae star densities, hence luminosities, by the Ritter relation $P\sqrt{\langle\rho\rangle} = Q$, and van Albada & Baker (1971) basic equations of stellar pulsation $P = f(M, L, T_{\rm eff})$, changes in the period mirror changes in the structure of the star, and thus are related to evolutionary effects. The period changes of the RR Lyrae stars thus represent a powerful tool for studying the evolution of the HB stars. In particular, period increases should indicate redward evolution, and decreases should indicate blueward evolution, while the rates of change can provide information about the timescales of the evolutionary phenomenon.

Evolutionary models (Sweigart & Renzini 1979; Lee, Demarque, & Zinn 1990) predict that the RR Lyrae variables in the Oo I clusters enter the instability strip as fundamental mode pulsators during their ZAHB phase and evolve from red to blue, therefore are expected to have decreasing periods. RR Lyrae variables in the Oo II clusters enter the strip as first overtone pulsators and evolve from blue to red, therefore are expected to have increasing periods. However, the theoretical evolutionary HB models do not adequately reproduce either the direction (increasing or decreasing period) or the rate of period changes observed in RR Lyrae stars, since both positive and negative changes are observed within RR Lyrae stars in a given cluster and, in absolute values, the observed rates of changes are sometime one or two orders of magnitude larger than predicted by evolutionary models. Iben & Rood (1970) concluded that both the size and the patterns of the period changes exclude a simple evolutionary explanation, as have others since (e.g. Clement et al. 1997, Alcock et al. 2000). Indeed, magnitudes of the observed period changes are surprisingly large. Period changes should not be seen so easily, yet they are, indicating that the evolution is a lot noisier than previously thought.

It has been argued (Lee 1991) that for a given cluster, it is the *mean rate* of period change that measures the evolutionary change, and that the *mean rate* of period change of RR Lyrae stars in globular clusters depends on horizontal branch morphology: clusters with extremely blue horizontal branches have predicted period increases occurring at a mean rate β=0.15 and up to 0.5 d per million years, while for clusters with red HBs the rate of the predicted period decrease is smaller (Lee 1991). However, one should be aware that in presence of noise even the mean rates of

change could be misleading.

Observations do not give definitive results. CC01 find that 30 of their M3 RR Lyrae variables lie above the ZAHB at intensity-averaged magnitude brighter than 15.55. They argue whether these stars are evolved objects. Sixteen of these overluminous stars have increasing periods, while nine have decreasing periods, and five have no previously determined period, thus tending to give some support the predictions of Lee (1991) models. However, of the stars lying above but closer to the ZAHB (V intensity-averaged magnitudes between 15.65 and 15.72) for which evolutionary models would predict blueward evolution (i.e. decreasing periods) 26 have increasing periods, 25 decreasing, two have constant period, and three have no previously determined period.

5.2.2. PERIOD CHANGES IN DOUBLE-MODE RR LYRAE STARS

Detection of changes in period in RR Lyrae stars usually require observations covering large time spans (of the order of decades), since variations are usually small and occur very slowly. However, changes in double-mode RR Lyrae stars seem to occur much faster than for single-mode stars, probably because these variables lie in the mode-switching region of the HB instability strip. Thus monitoring changes in period/mode of RRd's may require shorter time spans. Observational evidence exists that changes in double-mode RR Lyrae stars can occur on relatively short timescale: M68-V21 (Clement et al. 1993), M15-V31 (Purdue et al. 1995), and M3-V79 (Clement et al. 1997).

If an RRd star is evolving blueward, the amplitude of the first overtone should gain in strength relative to that of the fundamental. Vice versa, the strength of the fundamental mode oscillations should increase if the stars evolves redward. Analysis of the various data sets available so far for some of the M3 RRd's (Clement et al. 1997) show that the first overtone oscillations of both M3-V68 and M3-V79 have grown in strength since 1920s. For V79, the change is more striking. The analysis shows also that the fundamental period of V79 has decreased by a substantial amount in the last 35 years. Corwin et al. (1999) find a possible increase in the first overtone pulsation relative to the fundamental for M3-V99.

For the M3 RRd's for which so far there have been clear changes in the amplitude ratio, it is the strength of the first overtone pulsation that is increasing relative to the fundamental. Changes in the period and amplitude ratios of the double-mode RR Lyrae stars in the Oosterhoff type II cluster M15 are just the opposite than seen in the OoI cluster M3. This should indicate that the two clusters' double-mode variables are evolving in different directions on the horizontal branch, according to their different Oosterhoff types. However, period and mode changes observed in the Oosterhoff type II cluster M68 do not seem to fit into the above framework.

According to Purdue et al. (1995), the ratio of the first overtone to fundamental mode of pulsation has decreased in M15-V30 and possibly in M15-V53. Changes have occurred also in the amplitude ratio: the fundamental mode oscillations of M15-V30 have increased in strength relative to the first overtone between 1941 and 1991 with a rather abrupt change in the 1950s. The same

may be true of V53, though observational evidence is weaker. M15-V26 does not appear to be a double-mode variable in 1938, while it did show double-mode behavior with first overtone dominant mode in observations in 1951 (Jurcsik & Barlai 1990). All these period/mode changes are consistent with redward evolution.

Clement et al. (1993) found that the strength of the secondary (fundamental) mode of pulsation of M68-V21 was varying during the interval of their observations. The star presented only a primary first overtone period with no significant secondary pulsations in the analysis of its full data set (1986-1988 and 1989-1991 interval), while when the 1986-1988 data were analyzed alone they revealed pulsation in both the fundamental and first overtone modes, and that the amplitudes of the two modes were approximately equal. Walker (1994) found V21 to show double-mode behavior in observations taken in 1993. Thus it seems that double-mode pulsation in V21 is a transient phenomenon. M68-V33 also is reported to have changed from an RRd to RRc in the time span going from 1950 to 1986 (Clement et al. 1993). Walker (1994) confirms that this star is no longer an RRd. Thus the situation with the M68 RRd's is controversial and does not seem to be consistent with redward evolution.

A situation similar to M15-V30 may have occurred to the field RRd star AQ Leo, which has periods and period ratio similar to those of RRd stars in Oosterhoff type II systems like M15. Jerzykiewicz, Schult, & Wenzel (1982) found that the amplitude of the first overtone has decreased by an amount 0.012 ± 0.011, while the fundamental mode amplitude increased by 0.017 ± 0.011 in the time span from 1960 to 1974, with an abrupt increase of the first overtone period in the early 1970s. However this variable is not of much help in the present discussion on the connection between Oosterhoff types and direction of the HB evolution, since a field star cannot be associated with horizontal branch morphology, and so the directions of period changes cannot be so easily predicted.

5.2.3. MODE SWITCHING

Corwin et al. (1999) found that M3-V166 has switched its dominant pulsation mode from fundamental in 1992 to first overtone in 1993. In the present analysis we have found that also M3-V200 and M3-V251 have switched their dominant pulsation modes in the same time span. Mode switching is suggested as a possibility also for M15-V30 (Purdue et al. 1995) and for AQ Leo (Jerzykiewicz, Schult, & Wenzel 1982) but none of these two studies find direct evidence for the switch to have actually occurred in these stars. Statistics on mode switching are still very poor. Indeed, M3 remains so far the only stellar system where mode switching has been clearly observed. It is not clear whether this is an observational bias, and it is possible that further observations covering large time spans may reveal other mode switching RRd's among the double-mode RR Lyrae stars in M3 and other globular clusters. Together with new, better time-resolved theoretical models for RRd stars, the new data may help to understand whether mode switching is a natural phase in the evolution of double-mode stars expected theoretically, or it is, instead, just a further

manifestation of the "noise" that accompanies the HB evolution.

5.2.4. THE BLAZHKO PHENOMENON

Some connection very likely exists between period changes, switching of pulsation mode, double-mode pulsation and the Blazhko phenomenon (Blazhko 1907), a variation in both shape and amplitude of the light curve observed in about 30% of known Galactic RR Lyrae stars, that is superposed on the main period and which occurs with periodicities from a few days (e.g. 11 days, AH Cam, Smith et al. 1994) to a few hundreds of days (e.g. ~ 530 d, RS Bootis, Jones et al. 1988, and references therein).

Clementini et al. (1994) found that M4-V15, previously claimed to be an ab-type RR Lyrae (Sturch 1977, Cacciari 1979) in the photometric observations of 1986 showed a c-type light curve, and in 1989 an ab-type light curve with smaller amplitude and a different shape of the curve from previous studies. Clementini et al. (1994) suggested that V15 may be progressively changing its mode of pulsation, going from ab to c-type, while evolving towards the blue part of the RR Lyrae instability strip although they do not totally rule out that the star may be affected by a strong Blazhko effect. Walker (1994) found that seven of the fundamental mode RR Lyrae stars in M68, a cluster with a large number of RRd's, show the Blazhko effect, and only about half of the RR Lyrae stars in this cluster are singly periodic with stable light curves. Clement et al. (1997) found that prior to 1962, M3-V79 was an RRab star with an irregular light curve, a "Blazhko" variable, later became a double-mode pulsator. They suggested that this supports the explanation of the Blazhko effect in terms of mixing of pulsational modes. About 25% of the M3 RR Lyrae stars in CC01 sample have been recognized to be affected by the Blazhko effect by Cacciari et al. (2003).

5.2.5. PERIOD/MODE VARIATIONS AND THE DIRECTION OF THE HB EVOLUTION

Based on previous observational evidence Clement et al. (1997) concluded that changes in period and amplitude ratios of the double-mode RR Lyrae stars in M3 and M15 indicate that while OoI clusters like M3 are evolving blueward, Oo II systems like M15 may be evolving redward.

However, variations observed in the M68 RRd's do not seem to be consistent with this interpretation. Moreover, our new analysis shows that of the 3 newly discovered M3 double-mode RR Lyrae, M3-V200 and M3-V251 have both switched their dominant pulsation modes in the time span from 1992 to 1993, and while M3-V200 switched from fundamental to first overtone dominant mode similarly to M3-V166 (Corwin et al. 1999), in M3-V251 the mode switching has occurred in just the opposite way (i.e. from first overtone to fundamental dominant pulsation mode). This implies that if the evolutionary interpretation of the mode switching is correct, both blueward and redward evolution are occurring among the M3 double-mode RR Lyrae stars. Also, the observation of a switch in the dominant mode in less than 1 yr, as seen for V166, V200 and V251, is some-

how in contrast with the small rate in period change predicted for the Oo type I clusters by the evolutionary models.

We conclude that both extent and actual role of evolution on the period/mode variations of the RRd stars still remain unclear. On the other hand, if evolution is not the culprit of the mode switching observed in the M3 RRd's, as suggested by Purdue et al. (1995) in their concluding remarks, one must invoke some unknown instability which may be responsible for the changes in period ratios, amplitude ratios and/or dominant pulsation modes observed in M3 and M15.

6. SUMMARY AND CONCLUSIONS

Image subtraction analysis of the photometric data by CC01 has lead to recovering 15 RR Lyrae stars listed in Bakos et al. (2000) catalogue, but not found by CC01, and has resulted in improved periods for several other variables. We have identified three new double-mode RR Lyrae stars in M3 (V13, V200, and V251). A rough estimate of their masses has been obtained based on B01 pulsational models and the Petersen diagram. We find that both mass dispersion and strong evolutionary effects seem to be present among the RRd's, and the RR Lyrae stars in general, in M3. Two of the newly discovered RRd's have in fact period ratios as short as 0.738—0.739, and are well separated from all known RRd's in the Petersen diagram, at positions implying variations in mass by 0.1-0.2 $M_{\odot}$ and/or in the total heavy element content by a factor 2-2.5, among the M3 giants. However, given the rather homogeneous [Fe/H] abundance, the constant α-element enhancement, and the constant total [(C+N+O)/Fe] abundance derived for the M3 stars from several independent spectroscopic studies, the latter hypothesis seems rather unlikely.

Three out of the 8 M3 double-mode pulsators (V68, V79, and possibly V99) show variations in length and amplitude of the two pulsation periods. Moreover, Corwin et al. (1999) and the present study have shown for the first time that the M3 RRd variables V166, V200, and V251 have switched their dominant pulsation modes in a very short time-span (about one year) thus suggesting that these stars are undergoing a rapid evolutionary phase, and giving support to the evolutionary interpretation of the double-mode phenomenon.

Clear evidence for evolutionary effects occurring among the single-mode RR Lyrae stars in M3 has also been found recently by Cacciari et al. (2003) in their re-analysis of CC01 dataset. They found that about 25% of the M3 RR Lyrae stars in CC01 sample are affected by the Blazhko effect, and that 5% are more evolved than the average luminosity level of the M3 RR Lyrae stars.

Since changes in the pulsation characteristics of the M3 RRd's seem to occur on a much shorter time-scale than for single-mode RR Lyrae stars, RRd's may be a much more powerful tool to derive clues on the direction and rate of evolution along the HB, through the monitoring of their fast changes in period and pulsation modes.

However, drawing firm conclusions on the actual direction of the evolution on the M3 HB on the

basis of the present double-mode results seems premature here, given the small number of objects, the opposite behavior of V200 and V166 with respect to V251 (switching from fundamental to first overtone dominant modes, and vice versa, respectively), and the still rather short time base-line of the present observations. In this respect we recall that our results do not well fit into the scenario proposed by Clement et al. (1997) of the OoI clusters like M3 evolving blueward, and the OoII clusters like M15 evolving redward, since both blueward and redward evolution seems to occur among the M3 double-mode RR Lyrae stars.

A number of questions still remain open:

1 Is what we see in M3 a peculiarity of this cluster or are there other switching mode RRd stars that still lie undetected in globular clusters and in the general field?

2 Can the Petersen diagram be applied reliably to double-mode RR Lyrae stars undergoing rapid evolution to derive masses or other physical properties? Conversely, how does evolution affect the straightforward relation thought to exist between period and density (hence mass) via the Ritter and van-Albada and Baker equations? We know that evolutionary models are not adequate for interpreting the large changes in period observed in the RR Lyrae stars and their rates. Similarly, it may well be that the pulsational models and the "theoretical" Petersen diagram do not adequately predict masses for variable stars undergoing rapid evolutionary processes.

3 Is the anomalous spread in mass of the M3 RRd's real? If it is real, what mechanisms are causing it: mass-transfer in binary systems, helium enhancement, or, less likely, varying α-element enhancement among the M3 stars?

Additional data and continued monitoring of field and cluster RRd's over long time spans (decades) are needed to reveal changes in period and amplitude ratios, and mode switching, and to disentangle the role of evolution on the M3, M15 and M68 single and double-mode RR Lyrae variables and on the HB stars in general.

Elemental abundance analysis and dynamical studies are required to understand where the unusual masses of the M3 RRd's originated.

Radial velocity monitoring should also be undertaken to reveal possible binary systems and check whether mass transfer might be the cause of the low masses.

Finally, systematic searches with the image subtraction technique extending to the crowded cluster cores should be performed to reveal whether further RRd's still lie undetected in other Galactic globular clusters.

7. Appendix: Notes on Individual Stars

V13: This star is a candidate double-mode pulsator.

V29 and V155: In CC01 these two variables are interchanged as in Evstigneeva et al. (1994).

V44: This star has a very noisy light curve. Periods of about a half day and about one third day seem to phase the data reasonably well. Based on the shape of the light curve, we believe that it is and RRc variable and estimate the period at 0.33785. A period of about a half day was given in CC01 where it was listed as an RRab variable.

V180: This star was not found by CC01. In the web version of Clement et al. (2001) Catalogue of Variable Stars in Galactic Globular Clusters, available at http://www.astro.utoronto.ca/people.html, the star is classified as fundamental mode RR Lyrae, but no period is provided. The present data show that V180 is an ab-type RR Lyrae with period 0.61593 d.

V200: This star is a candidate double-mode pulsator.

V234: This star was misidentified in CC01 as V164. Our period for this star is 0.04073 d shorter than in Strader et al. (2002).

V239: This star was mistakenly identified in CC01 as an RRc variable with a period of 0.33343 d. This period is an alias of the correct period 0.66949 d. The shape of the light curve for V239 clearly identifies it as an RRab. This mistake was noted by Strader et al. (2002).

V242: The period we derive for this star is 0.05805 d shorter than in Strader et al. (2002).

V245: This star was misidentified in CC01 as V198.

V250: The period we derive for this star is 0.03171 d longer than in Strader et al. (2002).

V251: This star is a double-mode pulsator.

V252: This star has a very unusual light curve (see Figure 15). The new period is 0.05091 longer than in CC01.

V261: This star has very similar periods in CC01, the present paper and Strader et al. (2002, P$\sim$0.44 d). However, it is classified RRab by Strader et al. and RRc by CC01. The shape and amplitude of the light curve of V261 in Figure 1 of CC01 seem more consistent with a first overtone pulsator.

V270: The period we derive for this star is 0.19637 d longer than in Strader et al. (2002), and in good agreement with Clement et al. (2001) value of 0.69030 d.

We are grateful to Marcella Marconi for many helpful discussions and comments on the effects of α-enhancement and evolution on the theoretical Petersen diagram, to Carla Cacciari, Raffaele Gratton and Franca D'Antona for comments on the metallicity and helium spreads in M3 and

on the evolutionary effect in the M3 RR Lyrae stars, and to the referee J. Nemec for his helpful comments. BWC thanks the National Science Foundation for supporting this work through grant AST-9988156 to the University of North Carolina.

REFERENCES

Alard, C. 2000, A&AS, 144, 235

Alard, C. & Lupton, R. H. 1998, ApJ, 503, 325

Alcock, C. et al. (The MACHO Collaboration) 1996, AJ, 111, 1146

Alcock, C. et al. (The MACHO Collaboration) 1997, ApJ, 482, 89

Alcock, C. et al. (The MACHO Collaboration) 2000, ApJ, 542, 257

Armosky, B.J., Sneden, C., Langer, G.E., & Kraft, R.P. 1994, AJ, 108, 1364

Baker, R. H. & Baker, H. V. 1956, AJ, 61, 283

Bakos, B. A., Benko, J. M., & Jurcsik, J. 2000, Acta Astron., 50, 221

Barning, F.J.M. 1963, Bull. Astron. Inst. Netherlands, 17, 22

Blazhko, S. 1907, Astron. Nachr., 175, 325

Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1996, ApJ, 471, L33

Bragaglia, A., Gratton, R.G., Carretta, E., Clementini, G., Di Fabrizio, L., & Marconi, M. 2001, AJ, 122, 207 (B01)

Buonanno, R., Corsi, C. E., Buzzoni, A., Cacciari, C., Ferraro, F. R., & Fusi Pecci, F. 1994, A&A, 290, 69

Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1981, MNRAS, 196, 435

Cacciari, C. 1979, AJ, 84, 1542

Cacciari, C., Corwin, T.M., & Carney, B.W., 2003, in preparation

Caputo, F., Santolamazza, P., Marconi, M., 1998, MNRAS, 293, 364

Carney, B. W. 1996, PASP, 108, 900

Carretta, E., Cacciari, C., Ferraro, F. R., Fusi Pecci, F. & Tessicini, G. 1998, MNRAS, 298, 1005

Carretta, E., & Gratton, R.G. 1997, A&AS, 121, 95

Clement, C.M., Ferance, S., & Simon, N. 1993, ApJ, 412, 183

Clement, C.M., Hilditch, R. D., Kaluzny, J., & Rucinski, S. M. 1997, ApJ, 489, L55

Clement, C.M., Kinman, T. D., & Suntzeff, N. B. 1991, ApJ, 372, 273

Clement, C.M., et al. 2001, AJ, 122, 2587

Clementini, G., Carretta, E., Gratton, R., Merighi, R., Mould, J.R., McCarthy, J.K. 1995, AJ, 110, 2319

Clementini, G., et al. 2000, AJ, 120, 2054

Clementini, G., et al. 2003a, in Variable stars in the Local Group, eds. D. Kurtz & K. Pollard, ASP Conf. Series, in press

Clementini, G., Gratton, R.G., Bragaglia, A., Carretta, E., Di Fabrizio, L., & Maio, M. 2003b, AJ, 125, 1309 (astro-ph/0007471)

Clementini, G., Merighi, R., Pasquini, L., Cacciari, C., & Gouiffes, C. 1994, MNRAS, 267, 83

Corwin, T. M., Carney, B. W., & Allen, D. M. 1999, AJ, 117, 1332

Corwin, T. M. & Carney, B. W. 2001, AJ, 122, 3183 (CC01)

Cox, A. N., Hodson, S. W., & Clancy, S. P. 1983, ApJ, 266, 94

Cseresnjes, P. 2001, A&A, 375, 909

Dall'Ora, M., et al., 2003, AJ, in press (astro-ph/0302418)

D'Antona, F., Caloi, V., Montalban, J., Ventura, P., & Gratton, R. 2002, A&A, 395, 69

Demers, S., Irwin, M.J., Kunkel, W.E. 1994, AJ, 108, 1648

Evstigneeva, N. M.,Samus, N. N., Tsvetdova, T. M. & Shokin, Y. A. 1994, Pisma Astron. Zh., 20, 693

Ferraro, F. R., Carretta, E., Corsi, C. E., Fusi Pecci, F., Cacciari, C., Buonanno, R, Paltrinieri, B., & Hamilton, D. 1997a, A&A, 320, 757

Ferraro, F. R., Paltrinieri, B., Fusi Pecci, F., Cacciari, C., Dorman, B., & Rood, R. T. 1997b, ApJ, 484, L145

Ferraro, F. R., et al. 1997c, A&A, 324, 915

Garcia-Melendo, E., & Clement, C.M. 1997, AJ, 114, 1190

Goranskij, V. P. 1981, Inf. Bull. Variable Stars, 2007

Gratton, R.G., Bragaglia, A., Clementini, G., Carretta, E., Di Fabrizio, L., Maio, M., Taribello, E. 2003, in preparation

Gunn, J.E., & Griffin, R.F. 1979, AJ, 84, 752

Harris, W. E. 1996, AJ, 112, 1487

Hut, P. et al. 1992, PASP, 104, 981

Iben, I, Jr., & Rood, R.T. 1970, ApJ, 161, 587

Jerzykiewicz, M., & Wenzel, W. 1977, Acta Astron., 27, 35

Jerzykiewicz, M., Schult, R. H., & Wenzel, W. 1982, Acta Astron., 32, 357

Jones, R.V., Carney, B.W., & Latham, D.W. 1988, ApJ, 332, 206

Jurcsik, J., & Barlai, K., 1990, in Confrontation Between Stellar Pulsation and Evolution, eds. C. Cacciari & G. Clementini, ASP Conf. Series, Vol. 11, p. 112

Kaluzny, J., Hilditch, R. W., Clement, C., & Rucinski, S. M. 1998, MNRAS, 296,347

Kaluzny, J., Kubiak, M., Szymanski, M., Udalski, A., Krzeminski, W., Mateo, M. 1995, A&AS, 112, 407

Kovacs, G. 2001a, A&A, 375, 469

Kovacs, G. 2001b, in Stellar Pulsation-Nonlinear Studies, eds. M. Takeuti & D. D. Sasselov, Astrophysics and space science library, Vol. 257, (Kluwer Academic Publishers), 61

Kraft, R. P. & Ivans, I.I. 2003, PASP, 115, 143

Kraft, R. P., Peterson, R.C., Guhathakurta, P., Sneden, C., Fulbright, J.P., & Langer, G. E. 1999, ApJ, 518, L53

Kraft, R. P., Sneden, C., Langer, G. E., & Prosser, C.F. 1992, AJ 104, 645

Kraft, R. P., Sneden, C., Langer, G. E., Stetrone, M. D., & Bolte, M. 1995, AJ, 109, 2586

Lee Y.-W. 1991, ApJ, 367, 524

Lee S.-G. 1999, AJ, 118, 920

Lee Y.-W., Demarque P., & Zinn R. 1990, ApJ, 350, 155

Lomb, N.R. 1976, Ap&SS, 39, 447

Mackey, A.D., & Gilmore, G.F. 2003a, MNRAS, 338, 85

Mackey, A.D., & Gilmore, G.F. 2003b, MNRAS, 340, 175

Mackey, A.D., & Gilmore, G.F. 2003c, astro-ph/0307275

Mateo, M. 1998, ARAA, 36, 435

Mizerski, T. 2003, in Variable stars in the Local Group, eds. D. Kurtz & K. Pollard, ASP Conf. Series, in press

Moskalik, P., & Poretti, E. 2003, A&A, 398, 213

Nemec, J. M. 1985a, AJ, 90, 204

Nemec, J. M. 1985b, AJ, 90, 240

Nemec, J. M., & Clement, C.M. 1989, AJ, 98, 860

Nemec, J. M., Nemec, A. F. L. & Norris, J. 1986, AJ, 92, 358

Norris, J., Cottrell, P.L., Freeman, K.C., & Da Costa, G.S. 1981, ApJ, 244, 205

Norris, J., & Smith, G.H. 1984, ApJ, 287, 255

Oosterhoff, P. 1939, Observatory, 62, 104

Petersen, J.O. 1973, A&A, 27, 89

Pilachowski, C. A., & Sneden, C. 2001, AAS 199, 137.08

Popielski, B. L., Dziembowski, W. A., & Cassisi, S. 2000, Acta Astron., 50.491

Purdue, P., Silbermann, N.A., Gay, P., & Smith, H.A. 1995, AJ, 110, 1712

Pryor, C.P., Latham, D.W., & Hazen, M.L. 1988, AJ, 96, 123

Renzini, A. 1977, in Advanced Stages in Stellar Evolution, 7th Course of the Swiss Society of Astronomy and Astrophysics, Saas-Fee, p. 149, eds. P. Bouvier & A. Maeder, Geneva Observatory, Sauverny

Ripepi, V., et al. 2003, in Asteroseismology and Stellar Evolution, JENAM-2003 Symp., Budapest, Hungary, in press

Roberts, M. S. & Sandage, A. 1955, AJ, 60, 185

Rood, R. T., Carretta, E., Paltrinieri, B., Ferraro, F. R., Fusi Pecci, F., Dorman, B., Chieffi, A., Straniero, O., & Buonanno, R. 1999, ApJ, 523, 752

Rood, R. T., & Crocker, D. 1985, in Production and Distribution of C,N,O Elements, eds. J. Danziger, F. Matteucci, K. Kjar (ESO, Garching bei Munchen), p. 61

Salaris, M., & Cassisi, S. 1996, A&A, 305, 858

Sandage, A. 1953, AJ, 58, 61

Sandage A. 1959, ApJ, 129, 596

Sandage, A., Katem, B., & Sandage, M. 1981, ApJS, 46, 41

Scargle, J. D. 1992, ApJ, 263, 835

Smecker-Hane, T.A., Mandushev, G.I., Hesser, J.E., Stetson, P.B., Da Costa, G.S., & Hatzidimitriou, D. 1999, in Spectrophotometric Dating of Stars and Galaxies, eds. I. Hubeny, S. Heap & R. Cornett, ASP Conf. Series, Vol.192, p. 159 (astro-ph/9910212)

Smith, H. A. 1995, RR Lyrae Stars (Cambridge: Cambridge Univ. Press)

Smith, H. A., Matthews, J.M., Lee, K.M., Williams, J., Silbermann, N.A., & Bolte, M., 1994, AJ, 107, 679

Smith, G.H., Shetrone, M.D., Bell, R.A., Churchill, C.W., & Briley, M.M. 1996, AJ, 112, 1511

Smith, H. A., Silbermann, N., Baird, S., & Graham, J. 1992, AJ, 104, 1430

Soszyński, I., et al. 2002, Acta Astronomica, Vol. 52, 369

Soszyński, I., et al. 2003, astro-ph/0306041

Stellingwerf, R. F. 1978, ApJ, 224, 953

Strader, J., Everitt, H. O., & Danford, S. 2002, MNRAS, in press (astro-ph/0203135).

Sturch, C.R. 1977, PASP, 89, 349

Suntzeff, N.B. 1981, ApJS, 47, 1

Suntzeff, N.B. 1993, in The Globular Cluster-Galaxy Connection, eds. G.H. Smith & J.P. Brodie, ASP Conf. Series, Vol. 48, p. 14

Suntzeff, N.B., Schommer, R.A., Olszewski, E.W., & Walker, A. 1992, AJ, 104, 1743

Sweigart A.V., & Renzini A. 1979, A&A, 71, 66

Walker, A. R. 1992, AJ, 103, 1166

Walker, A. R. 1994, AJ, 108, 555

Walker, A. R., & Nemec, J. M. 1996, AJ, 112, 2026

Webbink, R.F. 1985, in Dynamics of Star Clusters, eds. J. Goodman & P. Hut, IAU Symp. 113, p. 541, Kluwer, Dordrecht

van Albada, T.S.,& Baker, N. 1971, ApJ, 169, 311

VandenBerg, D. 1992, ApJ, 391, 685

Zinn, R. & West, M. J. 1984, ApJS, 55, 45 (ZW84)

This preprint was prepared with the AAS LaTeX macros v5.0.

Fig. 1.— 1993 differential B flux light curves for the 15 variable stars not found in Corwin & Carney (2001). The data are for seven consecutive nights in 1993 and range from HJD 2449085 or 2449091. The order of the data is: filled squares, open squares, filled triangles, open triangles, filled circles, open circles, and crosses.

Fig. 2.— Metallicity distribution of all known double-mode RR Lyrae stars. Metal abundances are on the ZW84 and/or Clementini et al. (1995) metallicity scales (see Table 3, 4 and 5). These two scales are very similar. Only the 4 RRd's with known metal abundance are plotted of the MW field sample. Metallicities for the other galaxies refer to the whole system and not to individual RRd's. The open triangle with the arrow indicates the position in this diagram of the total number of the LMC field RRd's (Soszyński et al. 2003) assuming the average metal abundance of the LMC RR Lyrae in Clementini et al. (2003b) transformed to the ZW84 scale.

Fig. 3.— RR Lyrae specific frequency (S_{RR}; top panel) and horizontal-branch ratio (HBR; bottom panel) of the Galactic globular clusters (open circles) as a function of metal abundance. The 6 GGCs which host double-mode RR Lyrae are marked by filled circles. Also shown are the 4 extragalactic clusters which contain double-mode RR Lyrae stars (LMC: filled triangles, Fornax 3: filled pentagon). S_{RR}, HBR and [Fe/H] values are from Table 3.

Fig. 4.— Number of double-mode RR Lyrae stars in GCs versus: RR Lyrae specific frequency (S_{RR}, top panel), horizontal-branch ratio (HBR, middle panel), and cluster central concentration (c, bottom panel). The double c denotes core-collapsed clusters. Open and filled circles are used for the OoI and OoII Galactic clusters, respectively. Filled triangles are the LMC clusters, the filled pentagon is Fornax 3. Cluster parameters are given in Table 3.

Fig. 5.— Number of double-mode variables versus core radius (r_c, top panel), half-mass radius (r_h, middle panel), and tidal radius (r_t, bottom panel) of the cluster. Symbols are as in Figure 4.

Fig. 6.— Periodograms of the DAOPHOT V data of V13 obtained with GRATIS.

Fig. 7.— Light curves for V13 using the DAOPHOT V data.

Fig. 8.— Light curves of V13 using the DAOPHOT B data.

Fig. 9.— Periodograms for V200 using Alard B differential fluxes.

Fig. 10.— Light curves for V200, using Alard B differential fluxes.

Fig. 11.— Periodograms for V251 using Alard B differential fluxes.

Fig. 12.— Light curves for V251 using the Alard B differential fluxes.

Fig. 13.— Petersen diagram for all known double-mode RR Lyrae stars identified so far in various different stellar systems for which periods are available in the literature. Filled circles identify the three new candidate RRd's reported in this paper, filled square mark the previously

known M3 RRd's. Lines show the pulsational models by Bono et al. (1996), which correspond to Z=0.0001, and $M/M_{\odot}$=0.65, 0.75, and 0.80, and their extension to larger mass and metallicity ranges ($M/M_{\odot}$=0.85, and Z=0.0002, 0.0004, 0.0006, 0.0008) obtained by B01.

Fig. 14.— Enlargement of Figure 13 in the region relevant for the M3 RRd's. Lines corresponding to the different pulsational models are labelled by mass and luminosity level (in solar units).

Fig. 15.— 1993 differential B flux light curve for V252. Symbols as in Figure 1. Data are folded according ot the new period given in Table 2.

Table 1: Ephemerides for the variables recovered in the present analysis

Variable	Period (1)	Epoch (1)	Period (2)	Type (1)
V13	0.47951/0.35382	2448756.820	—	candidate RRd
V180	0.61593	2449091.787	—	RRab
V200	0.48674/0.35982	2449090.842	0.5292	candidate RRd
V210	0.35409	2449087.972	—	RRc
V242	0.59325	2449086.712	0.6513	RRab
V244	0.53786	2449090.809	—	RRab
V249	0.53301	2449087.724	—	RRab
V250	0.59031	2448753.775	0.5586	RRab
V251	0.47423/0.35384	2448756.820	—	candidate RRd
V253	0.33161	2449088.936	0.3328	RRc
V254	0.60455	2449089.769	0.6056	RRab
V255	0.57264	2449091.849	—	RRab
V257	0.60095	2449085.880	0.6019	RRab
V264	0.35649	2448755.093	0.3565	RRc
V270	0.69017	2448754.879	0.4938	RRab
V271	0.63043	2449088.870	0.6329	RRab

(1) Period, epoch of maximum light and type classification found by our analysis.
(2) Period from Strader et al. (2002).

Table 2. Variables with revised periods

Variable	New Period	CC01	CC01 diff.	Type
V8	0.63800	0.63916	-0.00116	RRab
V44	0.33785	0.50635	-0.16850	RRc, noisy curve
V115	0.51258	0.51335	-0.00077	RRab
V122	0.51609	0.50603	0.01006	RRab
V148	0.46729	0.46597	0.00132	RRab
V149	0.54815	0.54996	-0.00181	RRab
V157	0.54282	0.54187	0.00095	RRab
V159	0.53326	0.53370	-0.00044	RRab
V161	0.52164	0.51444	0.00720	RRab
V165	0.48363	0.48504	-0.00141	RRab
V168	0.27597	0.27643	-0.00046	RRc
V170	0.43226	0.43569	-0.00343	RRc
V174	0.58703	0.59450	-0.00747	RRab
V175	0.56970	0.56584	0.00386	RRab
V181(H903)	0.66384	0.66790	-0.00406	RRab
V184	0.53213	0.52958	0.00255	RRab
V189	0.61294	0.61869	-0.00575	RRab
V191	0.52003	0.51921	0.00082	RRab
V192	0.49790	0.48145	0.01645	RRab
V193	0.74784	0.73285	0.01499	RRab
V201	0.54141	0.53963	0.00178	RRab
V207	0.34458	0.34494	-0.00036	RRc
V208	0.33802	0.33735	0.00067	RRc
V209	0.34939	0.34718	0.00221	RRc
V214	0.54045	0.53949	0.00096	RRab, Blazhko
V215	0.52710	0.53308	-0.00598	RRab
V218	0.54487	0.54308	0.00179	RRab
V219	0.61366	0.61139	0.00227	RRab
V220	0.60011	0.59585	0.00426	RRab
V222	0.50151	0.50065	0.00086	RRab
V226	0.48843	0.48769	0.00074	RRab
V234(V164)	0.50877	0.50959	-0.00082	RRab
V235	0.76519	0.76175	0.00344	RRab
V239(X13)	0.66949	0.33343	0.33606	RRab
V240(X14)	0.27601	0.27647	-0.00046	RRc
V241(X17)	0.59408	0.59617	-0.00209	RRab
V243(X22)	0.62991	0.63220	-0.00229	RRab
V245(V198)	0.28403	0.28305	0.00098	RRc
V246(X30)	0.33914	0.33845	0.00069	RRc
V248(X36)	0.51134	0.51065	0.00069	RRab
V252(KG4)	0.50155	0.45064	0.05091	?

Table 2—Continued

Variable	New Period	CC01	CC01 diff.	Type
V259(KG15)	0.33287	0.33352	-0.00065	RRc
V261(H812)	0.44414	0.44467	-0.00053	RRc

Table 3: Informations on the RRd variables identified in globular clusters

ID-Name	N_{RR} (a)	N_{RRd}	[Fe/H] (b)	Oo (c)	S_{RR} (d)	HBR (e)	c (f)	$r_c(')$ (f)	$r_h(')$ (f)	$r_t(')$ (f)	$< M/M_{\odot} >$ (g)	σ_M (g)
						Milky Way						
IC4499	97	16	-1.50	I	113.4	0.11	1.11	0.96	1.50	12.35	0.853	0.023
NGC5272-M3	182	5+3	-1.66	I	49.0	0.08	1.84	0.55	1.12	38.19	0.779	0.063
NGC4590-M68	42	12	-2.09	II	48.3	0.17	1.64	0.69	1.55	30.34	0.755	0.019
NGC2419	31	1	-2.10	II	4.6	0.86	1.40	0.35	0.73	8.74	0.793	–
NGC7078-M15	88	14	-2.15	II	18.9	0.67	2.50	0.07	1.06	21.50	0.785	0.036
NGC6426	14	1	-2.20	II	25.3	0.58	1.70	0.26	0.96	13.23	0.814	–
						Large Magellanic Cloud						
Reticulum	32	4	-1.71	I	132.2	-0.04±0.05	1.0	1.0	1.58	10.0	–	–
NGC1835	84	6	-1.79	I/II	18.2	–	1.5	0.10	0.32	3.16	–	–
NGC2019	41	2	-1.81	–	27.3	–	1.6±0.3	0.07±0.03	0.24	2.63	–	–
						Fornax						
Fornax 3	99	8	-1.91	II/(I)	90.2	0.40±0.05	1.28	0.07 ± 0.02	0.16	1.28±0.08	–	–
							1.83	0.02 ± 0.01	0.12	1.58±0.37		

[a] N_{RR} values are from Clement et al. (2001) for the MW clusters, from Walker (1992) and Ripepi et al. (2003) for the Reticulum, from Soszyński et al. (2003) for NGC1835 and NGC 2019, from Mackey & Gilmore (2003c) for Fornax 3.
[b] Metall abundances for the MW clusters are from ZW84. Metallicities for the LMC clusters are taken from Suntzeff et al. (1992), and from Clementini et al. (2003a) for Fornax 3. They are on the ZW84 metallicity scale.
[c] The Oosterhoff type classification (Oo) for the Galactic clusters is taken from Clement et al. (1991), for the Reticulum cluster is from Walker (1992) and Ripepi et al. (2003), for NGC 1835 is from Soszyński et al. (2003), and for Fornax 3 from Clementini et al. (2003a).
[d] The RR Lyrae specific frequency for the Milky Way clusters is from the 2003 update of Harris (1996) web catalogue on GGCs. For the LMC clusters S_{RR} was computed from the N_{RR} values in column 2 adopting M_V values from Suntzeff et al. (1992), for Fornax 3 is from Mackey & Gilmore (2203c).
[e] The HBR value of the MW clusters is from Harris catalogue, for Reticulum is from Walker (1992), for Fornax 3 is from Mackey & Gilmore (2203c).
[f] Structural parameters for the MW clusters are from Harris catalogue. c, r_c, r_t values for the LMC clusters are from Suntzeff et al. (1992), for Fornax 3 are from Demers et al. (1994) and Webbink (1985; see also Mackey & Gilmore 2003a,b, for new estimates of the core radius of NGC 1835, NGC 2019, and Fornax 3). Half mass radii (r_h) for the extragalactic clusters have been computed according to the definition given in Harris catalogue ($log(r_h/r_c) = 0.6\times c - 0.4$).
[g] Average masses and σ_M values of the MW globular cluster RRd's have been computed from the values published in B01. In this paper the interested reader can find individual pulsational masses for the Galactic cluster RRd's and for some of the field RRd's in the MW and in the LMC.

Table 4: Informations on the field RRd variables found in Local Group galaxies

ID-Name	$N_{\rm RRd}$	[Fe/H][a]	Oo	r_c^b (arcmin)	r_c^b (pc)	r_t^b (arcmin)	Ref. (for [Fe/H])
Milky Way	22[c]	–	I[d]	–	–	–	–
LMC	230	-1.54	I/II	–	–	–	1
SMC	57	-1.7	I/II	–	–	–	2
Sagittarius	40	-1.6	I/(II)	–	550	$> 10\,$deg	3
Sculptor	18	-1.80	I/II	5.8±1.6	110	76.5±5.0	4
Fornax	12	-1.77	I/II	13.8±0.8	460	71±4	5
Draco	10	-2.00	II/I	9.0±0.7	180	28.3±2.4	4
Carina	6	-2.1	II/I	8.8±1.2	210	28.8±3.6	6

[a]Metall abundances are for the whole system and not for individual RRd's, they are on or were transformed to the ZW84 metallicity scale.
[b]Structural parameters for the dwarf galaxies are taken from Mateo (1998).
[c]The number of MW RRd's includes 6 previously knonw objects (see Clementini et al. 2000), 3 new discoveries in the galactic bulge (Mizerski 2003), and 13 RRd's lying along the line of sight to Sagittarius (Cseresnjes 2001).
[d]The Oosterhoff type classification for the Milky Way refers only to variables in the Galactic Center (Cseresnjes 2001).
References.- (1) Clementini et al. (2003b) transformed to the ZW84; (2) Smith et al. (1992); (3) Cseresnjes (2001); (4) Mateo (1998); (5) Clementini et al. (2003a); (6) Smecker-Hane et al. (1999).

Table 5: Individual metallicities of field RRd variables

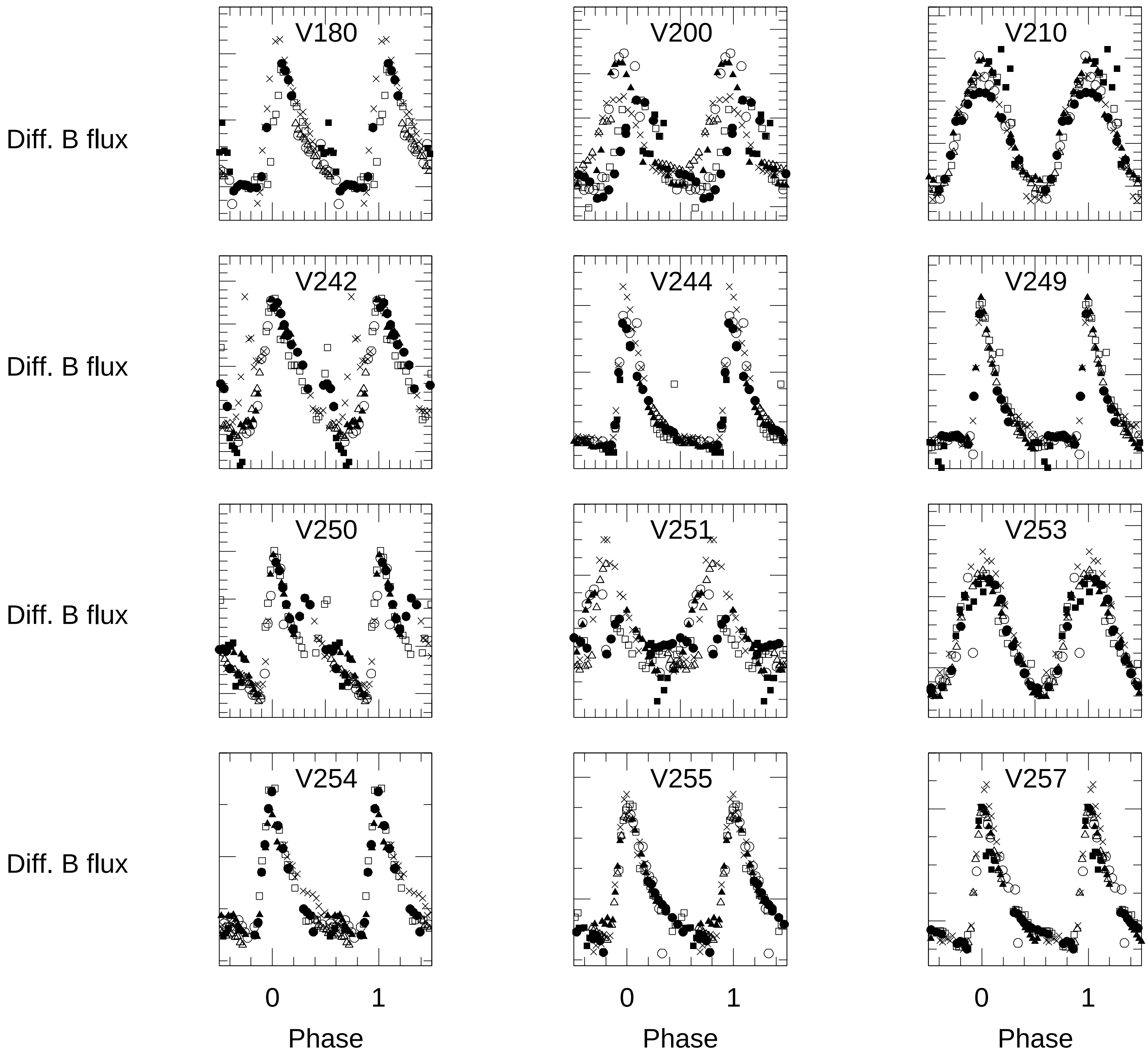
ID-Name[a]	[Fe/H][b]	Ref.
MW-RRVIII-58	-1.75±0.20	1
MW-AQLeo	-1.81±0.20	1
MW-RRVIII-10	-1.86±0.20	1
MW-CUCom	-2.38±0.20	2
LMC-CA48 - 06.6811.651	-1.38±0.13	3
LMC-CB61 - 13.5958.518	-1.23±0.20	1
LMC-CB49 - 13.5836.525	-1.71±0.13	3
LMC-CB45 - 13.6080.591	-1.70±0.10	3
LMC-CA02 - 13.6691.4052	-1.74±0.20	1
- 06.6691.1003		
LMC-CA67 - 06.6810.428	-2.07±0.13	3
LMC-CB59 - 13.5838.497	-1.54±0.09	3
LMC-CA05 - 13.7054.2970	-1.60±0.27	3
LMC-CA12 - 06.6933.939	-1.42±0.14	3

[a]For the LMC variables we list both B01 and Alcock et al. (1997) identification numbers.
[b]Metall abundances are either on the ZW84 and/or on Clementini et al. (1995) metallicity scales. These two scales are very similar. Metallicities for 7 of the LMC RRd's were taken from Gratton et al. (2003, in preparation) and transformed to the ZW84 scale according to the procedure described in Clementini et al. (2003b).
References.- (1) B01; (2) Clementini et al. (2000); (3) Gratton et al. (2003, transformed to the ZW84 metallicity scale).

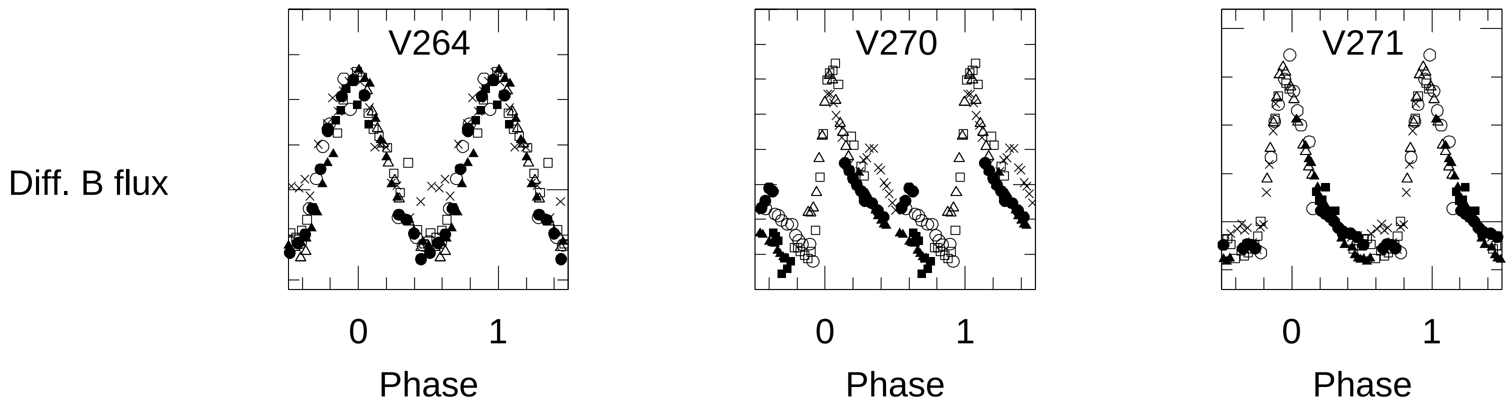
Table 6: Parameters of the RRd variables in M3

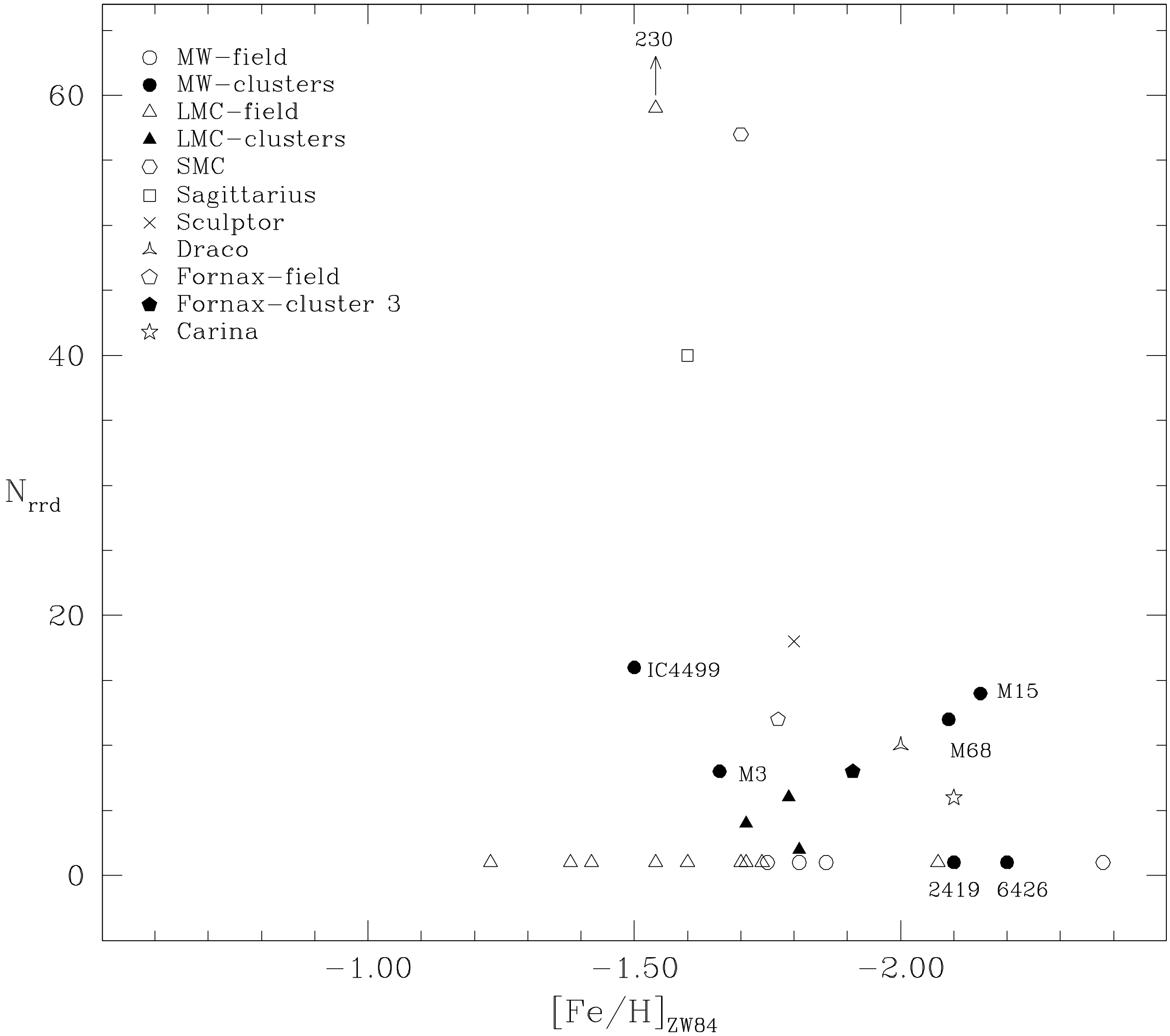
Variable	P_0	P_1/P_0	$M/M_\odot$	$< V_{int} >$ (1)	$< B_{int} > - < V_{int} >$ (1)	r (arcsec)
V68	0.479	0.7432	0.68±0.11	15.642	0.288	186.9
V79	0.4797	0.7453	0.77±0.14	15.658	0.278	362.7
V87	0.48	0.7458	0.80±0.15	15.578	0.274	134.4
V166	0.482	0.7459	0.80±0.15	15.700	0.289	92.8
V99	0.4835	0.7468	0.85±0.16	15.602	0.276	210.0
V251	0.47423	0.7461	$\sim$ 0.8	–	–	3.7
V200	0.4867	0.739	$\sim$ 0.65	–	–	32.0
V13	0.47951	0.7379	$\sim$ 0.55/0.65	15.612	0.270	129.1

(1) Intensity-averaged magnitudes and colors are taken from CC01, they are before pre-whitening. The values after pre-whitening derived for V13 in the present analysis are 15.662 and 0.268, respectively.

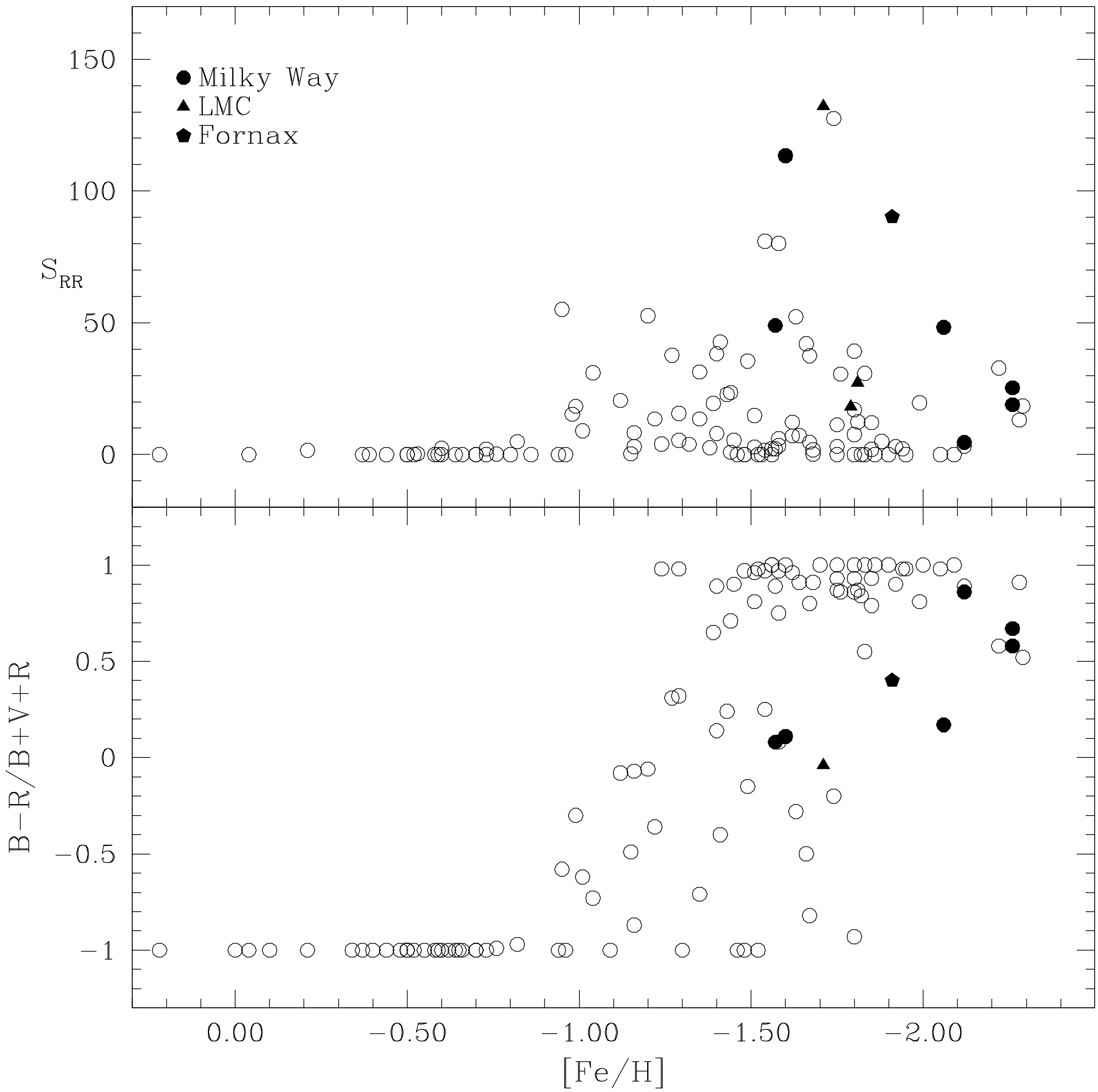


V180
V200
V210
V242
V244
V249
V250
V251
V253
V254
V255
V257
Diff. B flux
Diff. B flux
Diff. B flux
Diff. B flux
0
1
0
1
0
1
Phase
Phase
Phase

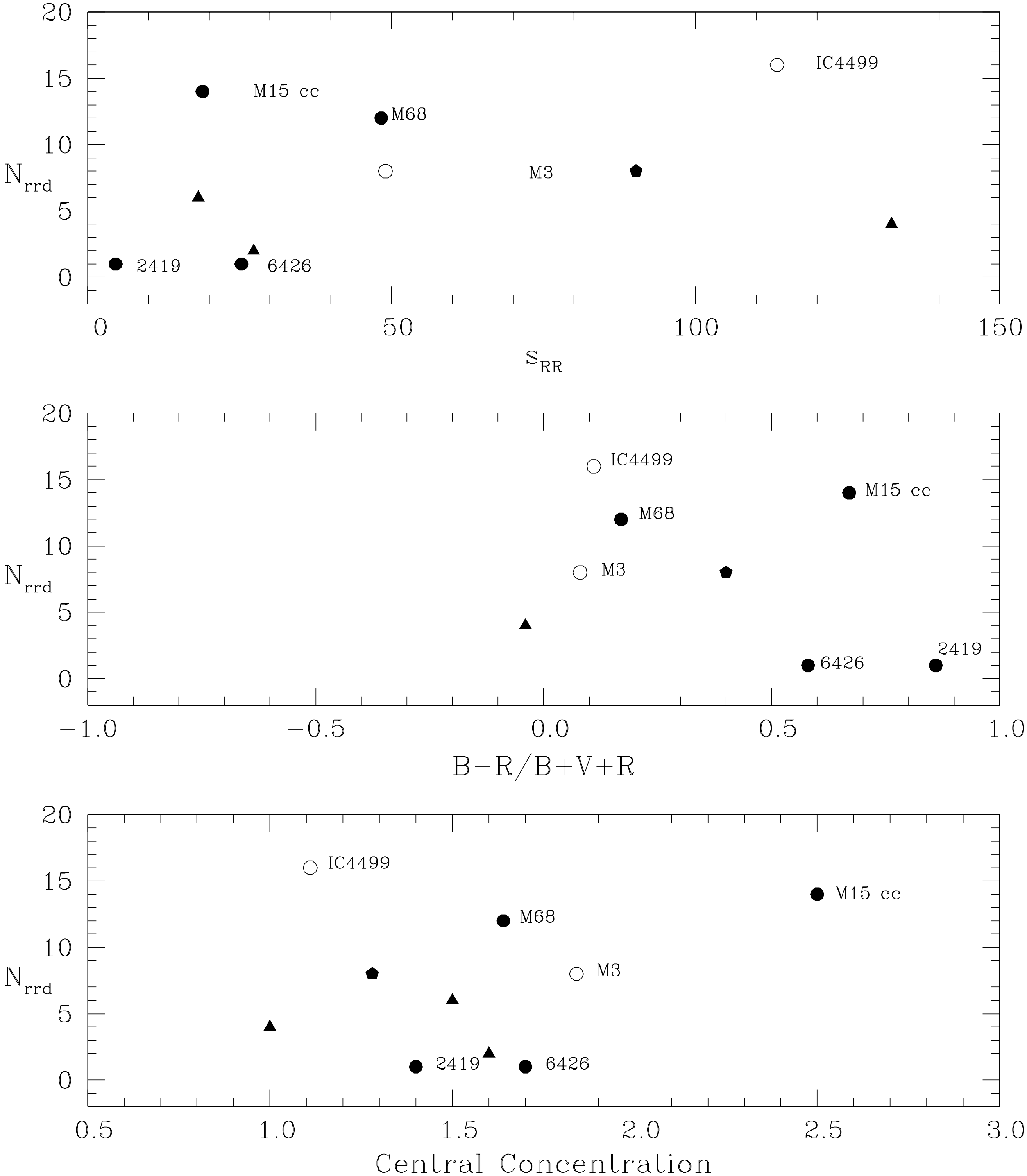

Diff. B flux
V264
V270
V271
0
1
Phase

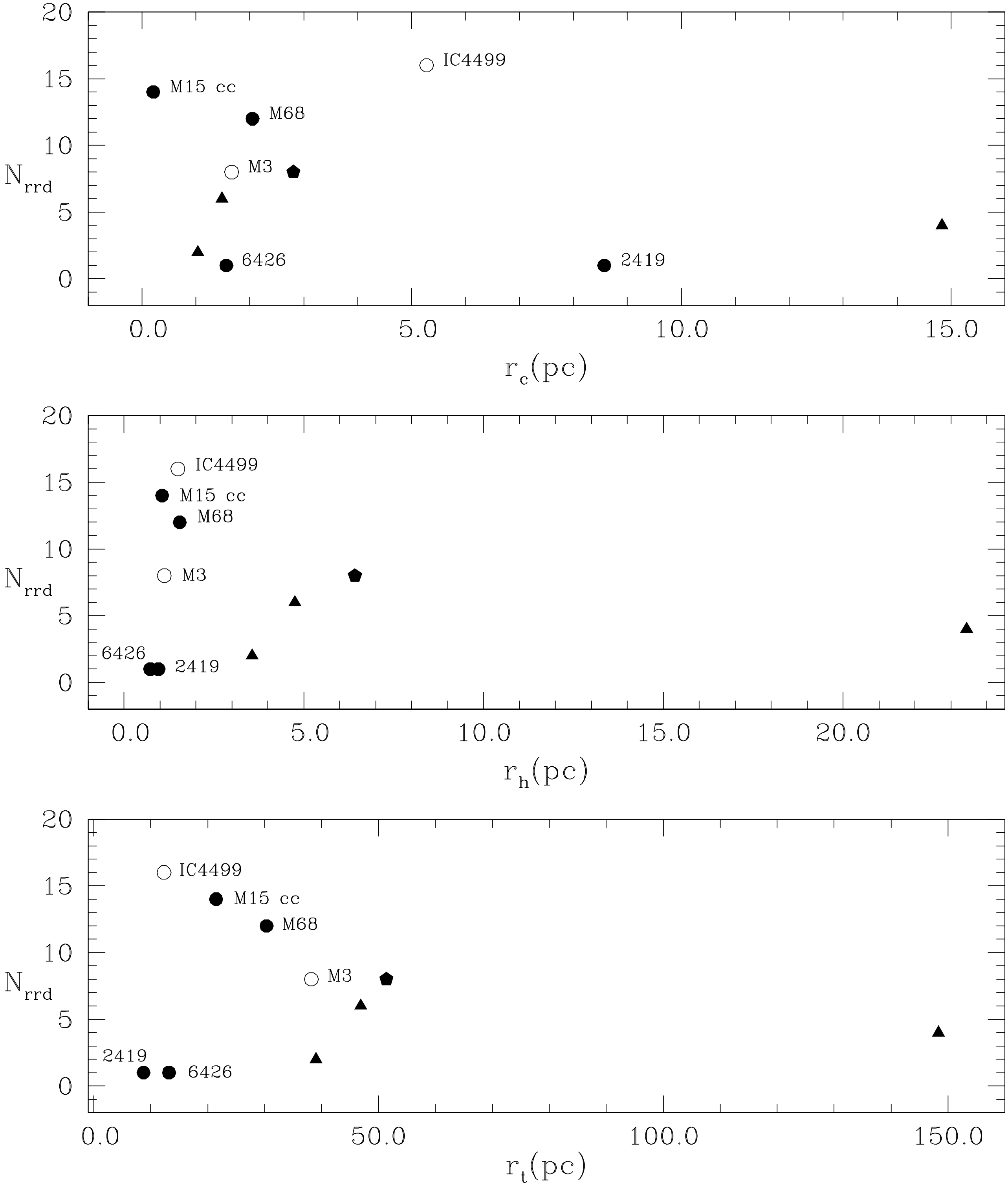


MW−field
MW−clusters
LMC−field
LMC−clusters
SMC
Sagittarius
Sculptor
Draco
Fornax−field
Fornax−cluster 3
Carina
230
IC4499
M3
M15
M68
2419
6426
60
40
20
0
−1.00
−1.50
−2.00
N_{rrd}
$[Fe/H]_{ZW84}$

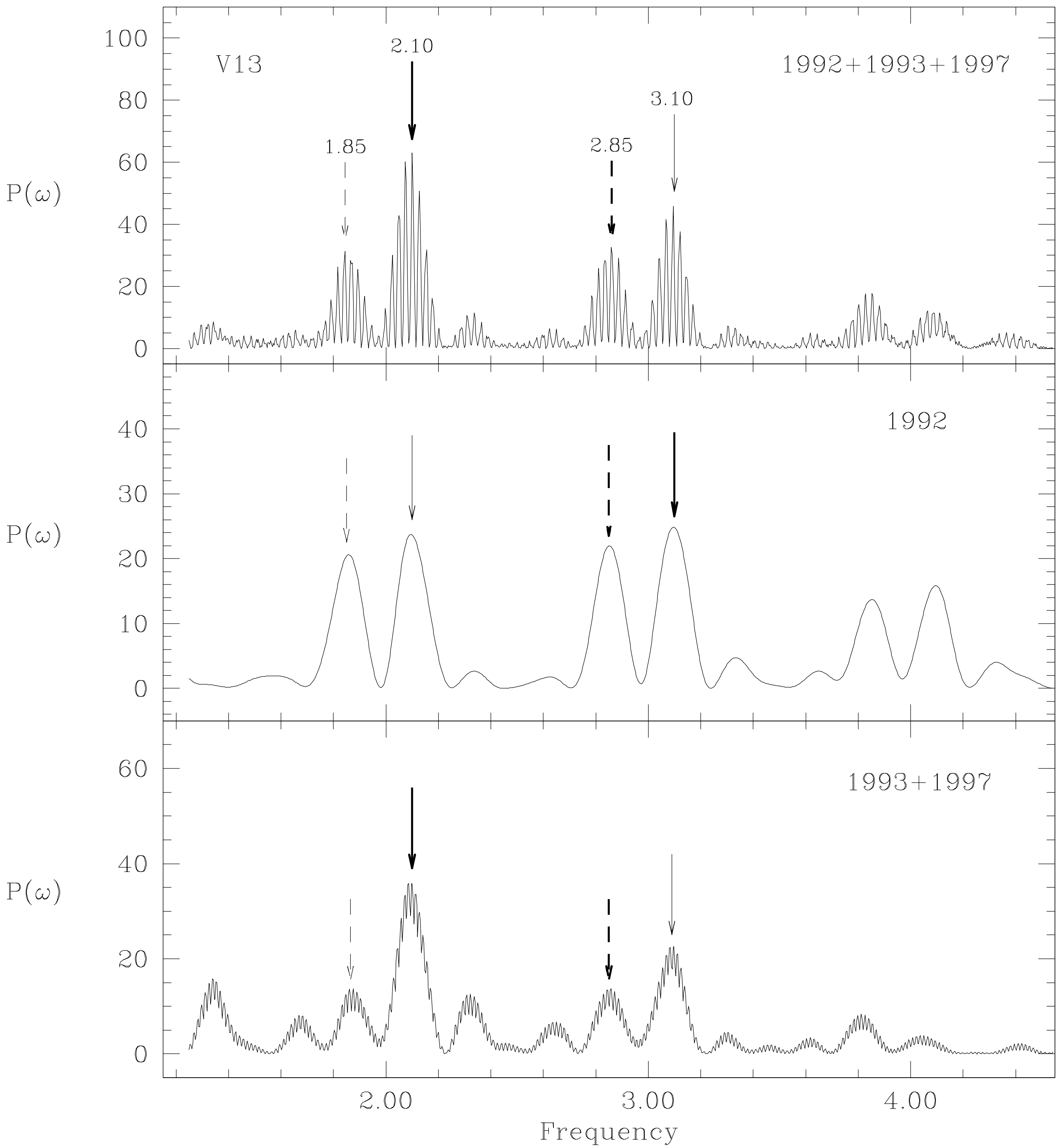


Milky Way
LMC
Fornax
S_{RR}
150
100
50
0
1
0.5
0
−0.5
−1
B−R/B+V+R
0.00
−0.50
−1.00
−1.50
−2.00
[Fe/H]

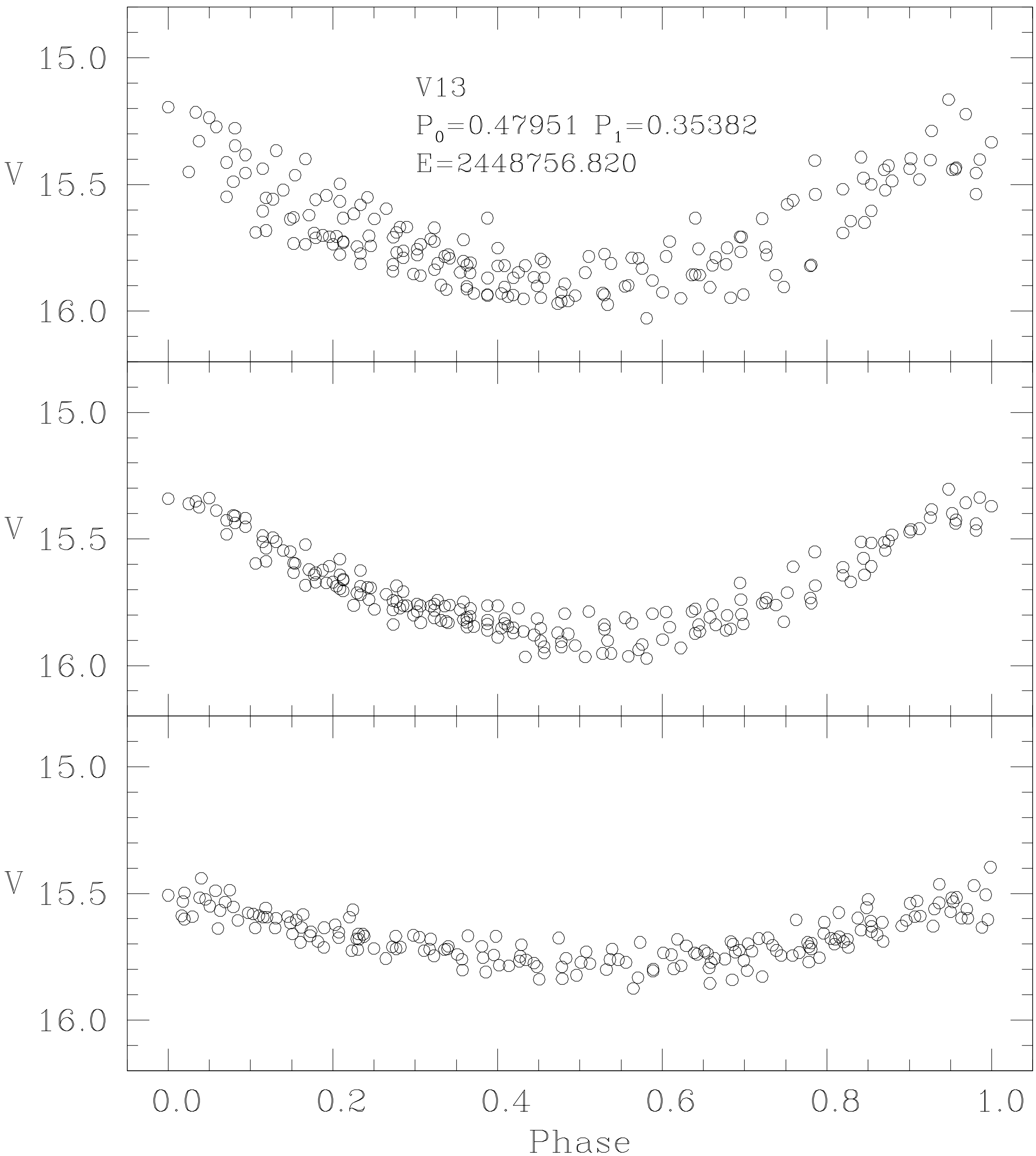

20
15
10
5
0
N_rrd
M15 cc
M68
IC4499
M3
2419
6426
0
50
100
150
S_RR
−1.0
−0.5
0.0
0.5
1.0
B−R/B+V+R
0.5
1.0
1.5
2.0
2.5
3.0
Central Concentration



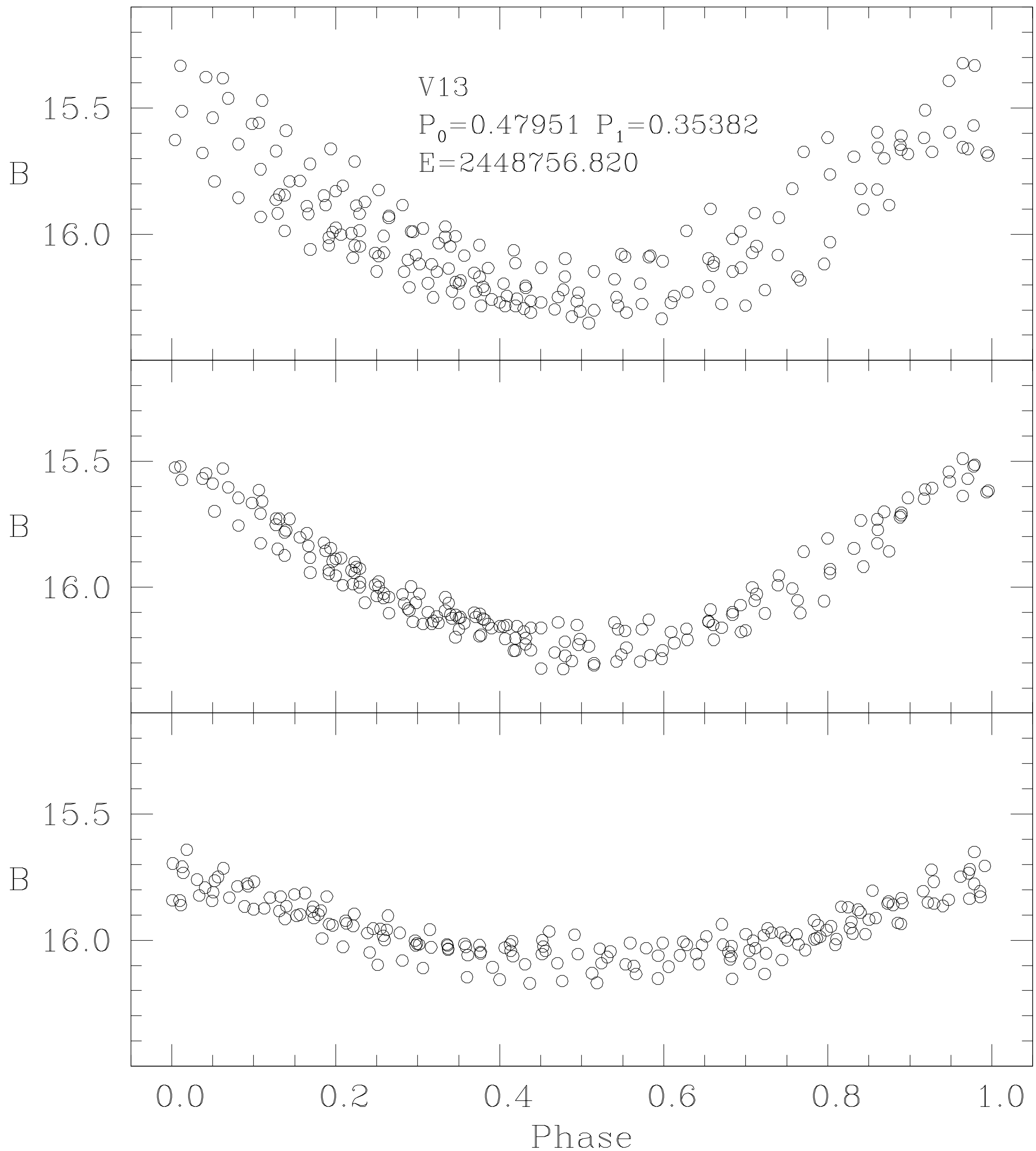
20
15
10
5
0
N_{rrd}
IC4499
M15 cc
M68
M3
6426
2419
0.0
5.0
10.0
15.0
r_c(pc)
20.0
r_h(pc)
50.0
100.0
150.0
r_t(pc)



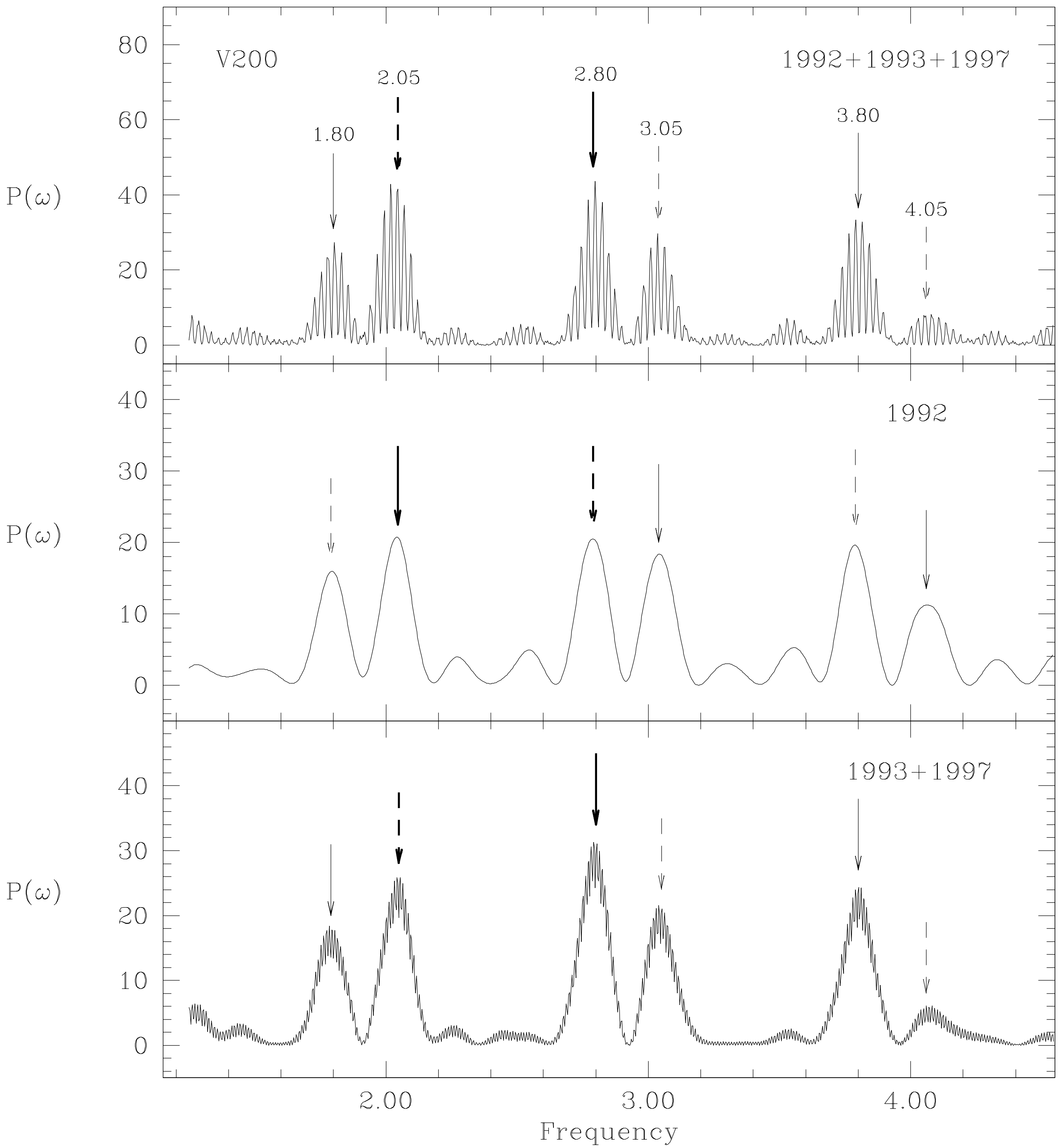
V13
1992+1993+1997
2.10
1.85
2.85
3.10
1992
1993+1997
P(ω)
Frequency
2.00
3.00
4.00



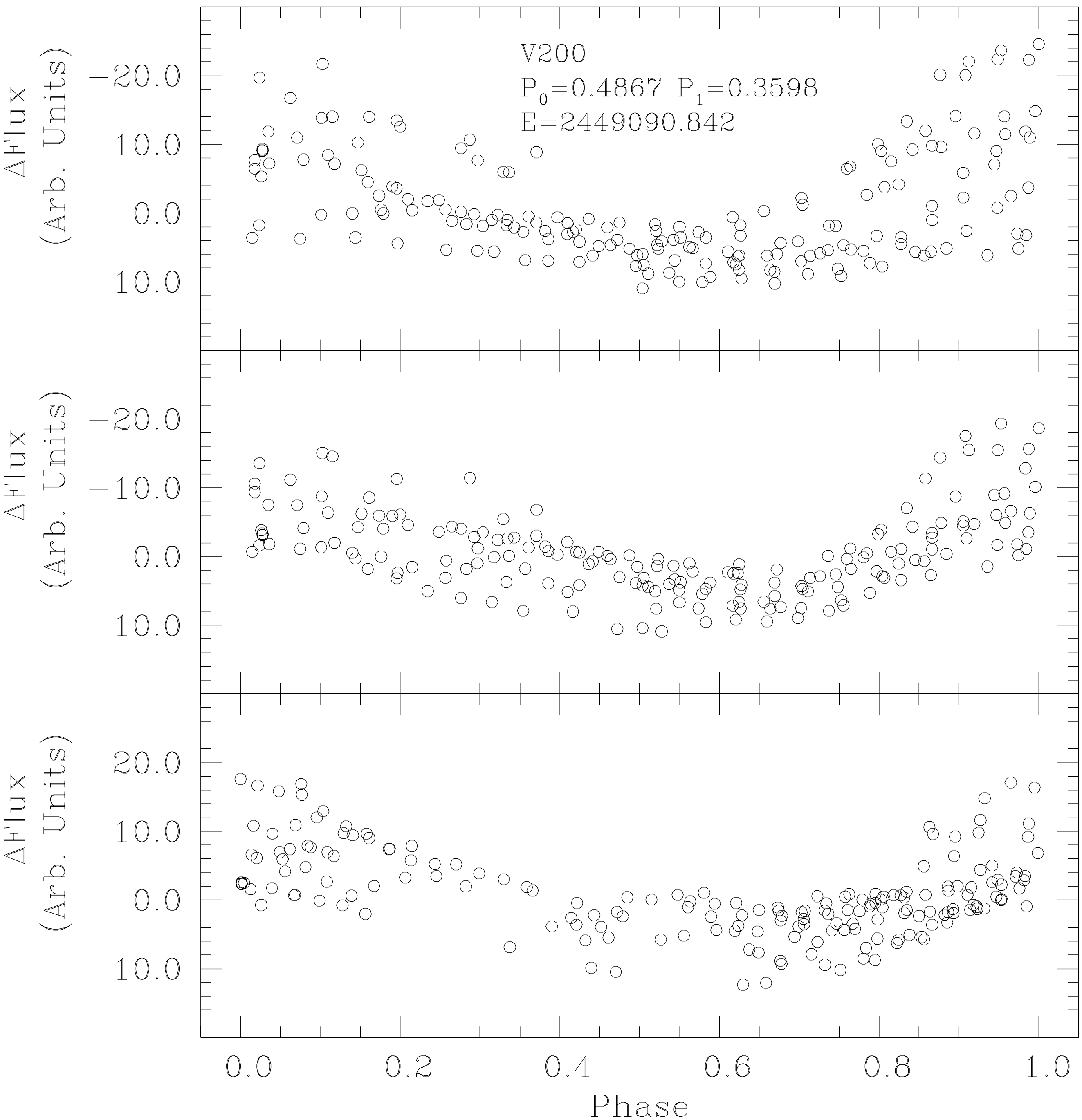
V13
P_0=0.47951 P_1=0.35382
E=2448756.820
V
15.0
15.5
16.0
0.0
0.2
0.4
0.6
0.8
1.0
Phase

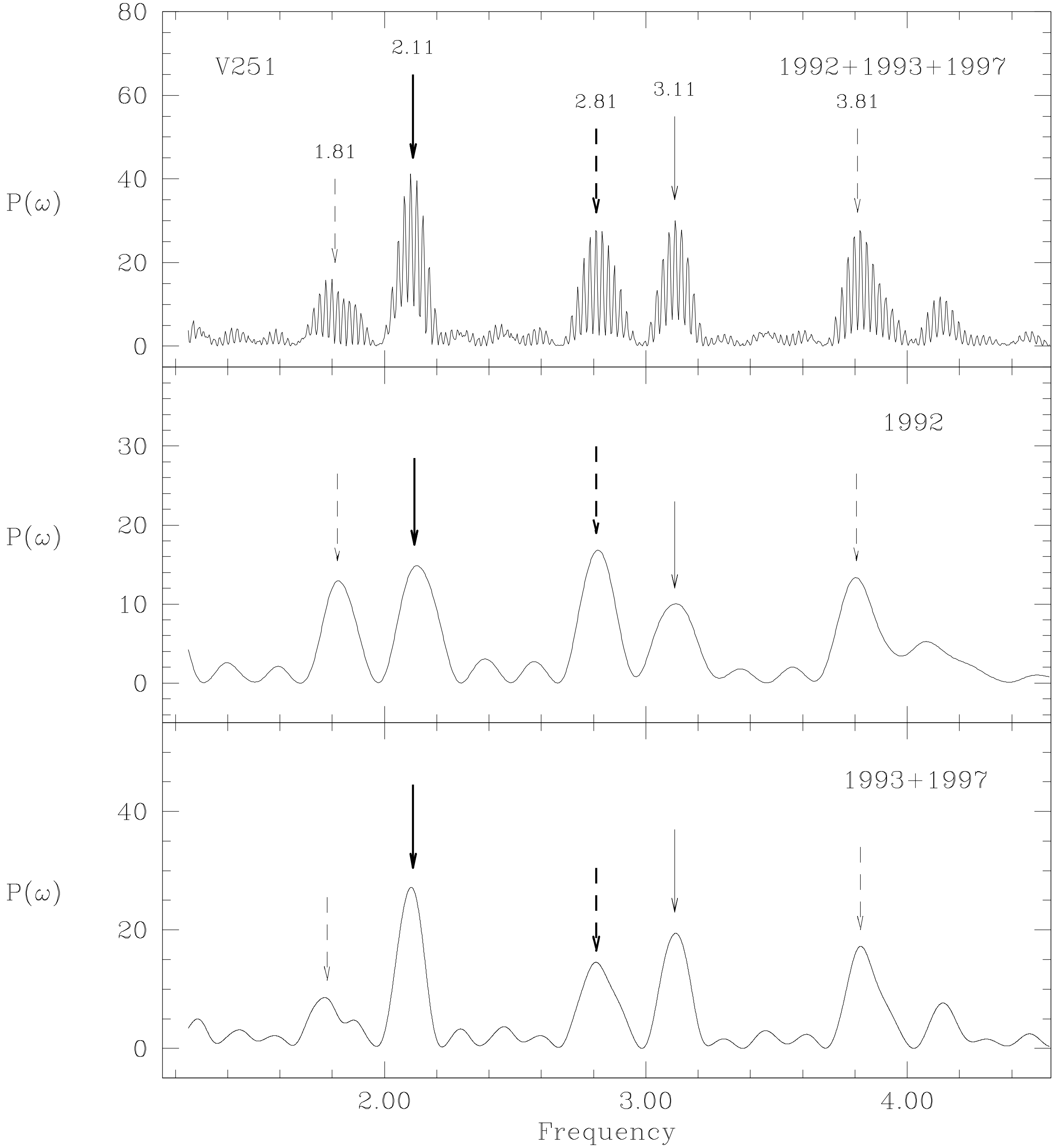


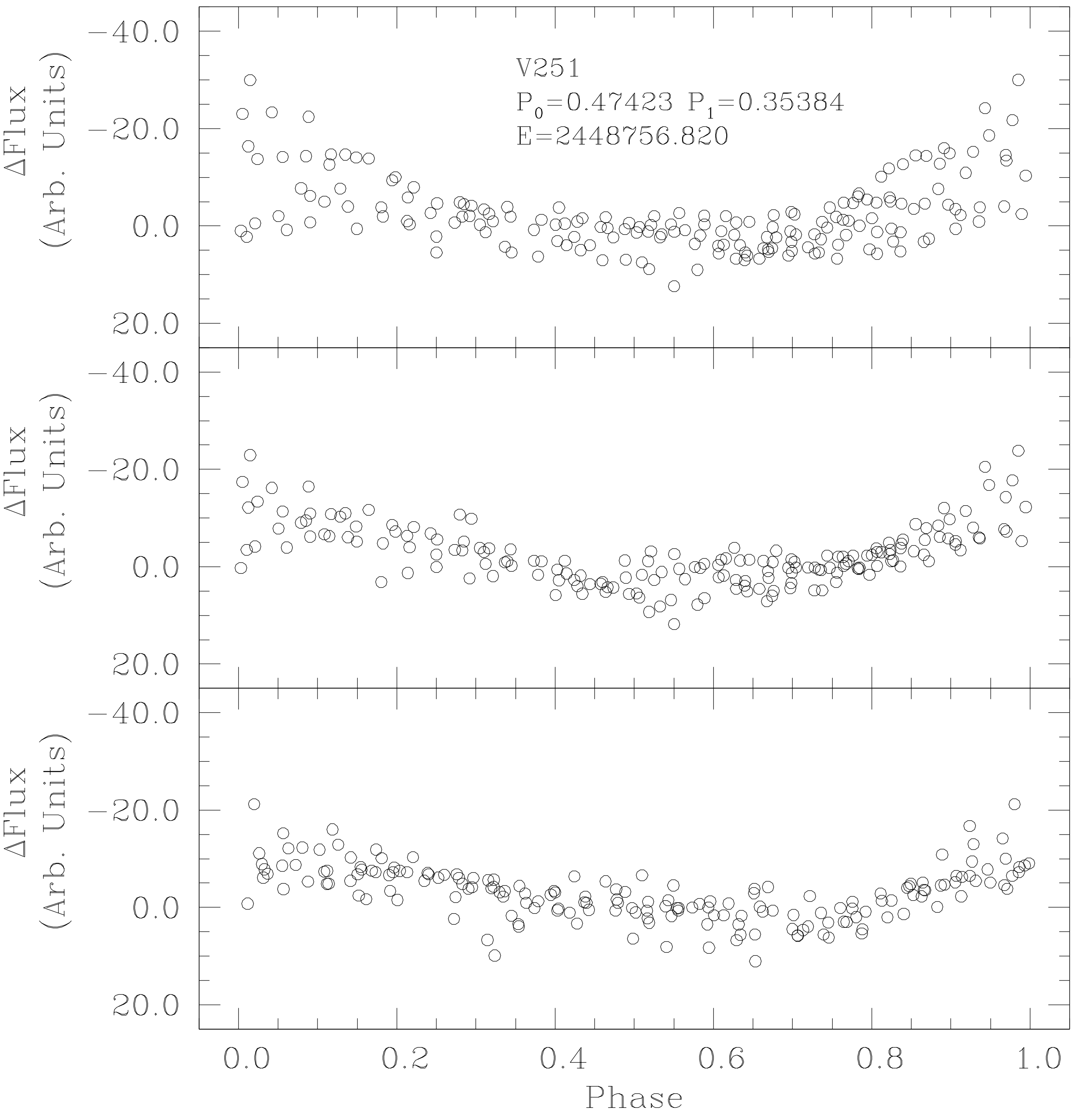
V13
P_0=0.47951 P_1=0.35382
E=2448756.820
B
15.5
16.0
0.0
0.2
0.4
0.6
0.8
1.0
Phase



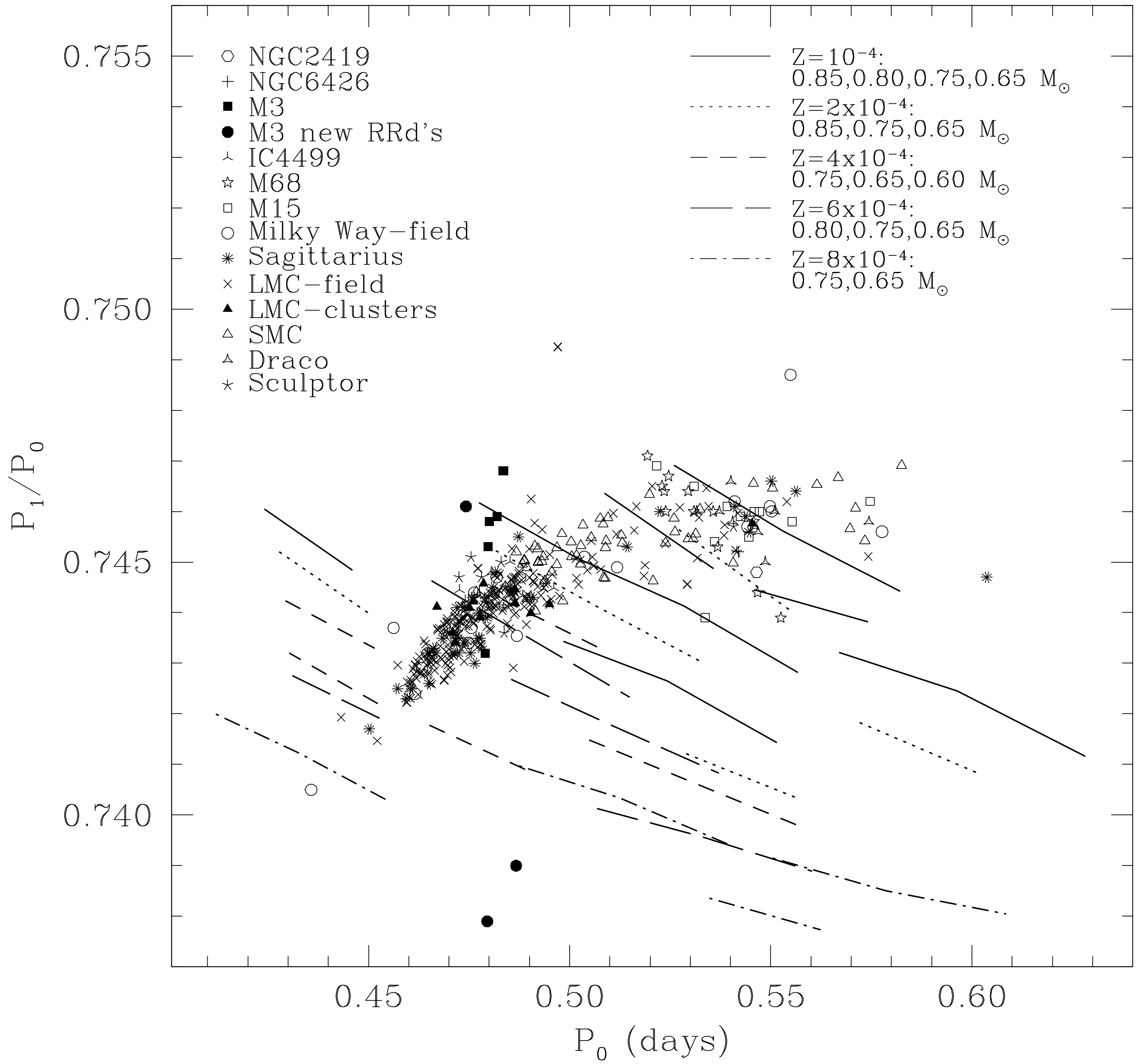
V200
1992+1993+1997
1.80
2.05
2.80
3.05
3.80
4.05
1992
1993+1997
P(ω)
Frequency

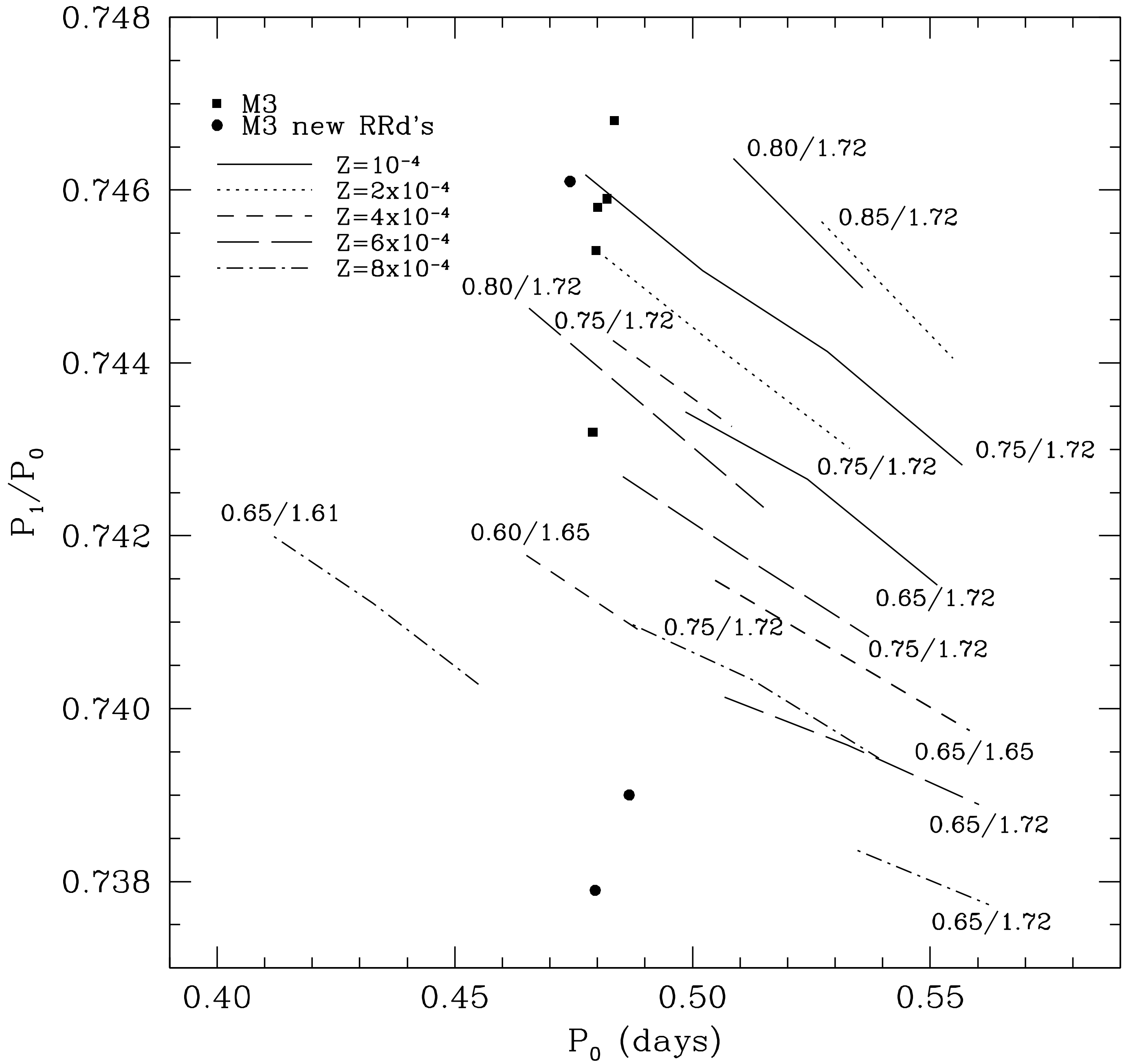

V200
P_0=0.4867 P_1=0.3598
E=2449090.842
ΔFlux
(Arb. Units)
−20.0
−10.0
0.0
10.0
0.0
0.2
0.4
0.6
0.8
1.0
Phase


V251
1992+1993+1997
2.11
1.81
2.81
3.11
3.81
1992
1993+1997
P(ω)
Frequency

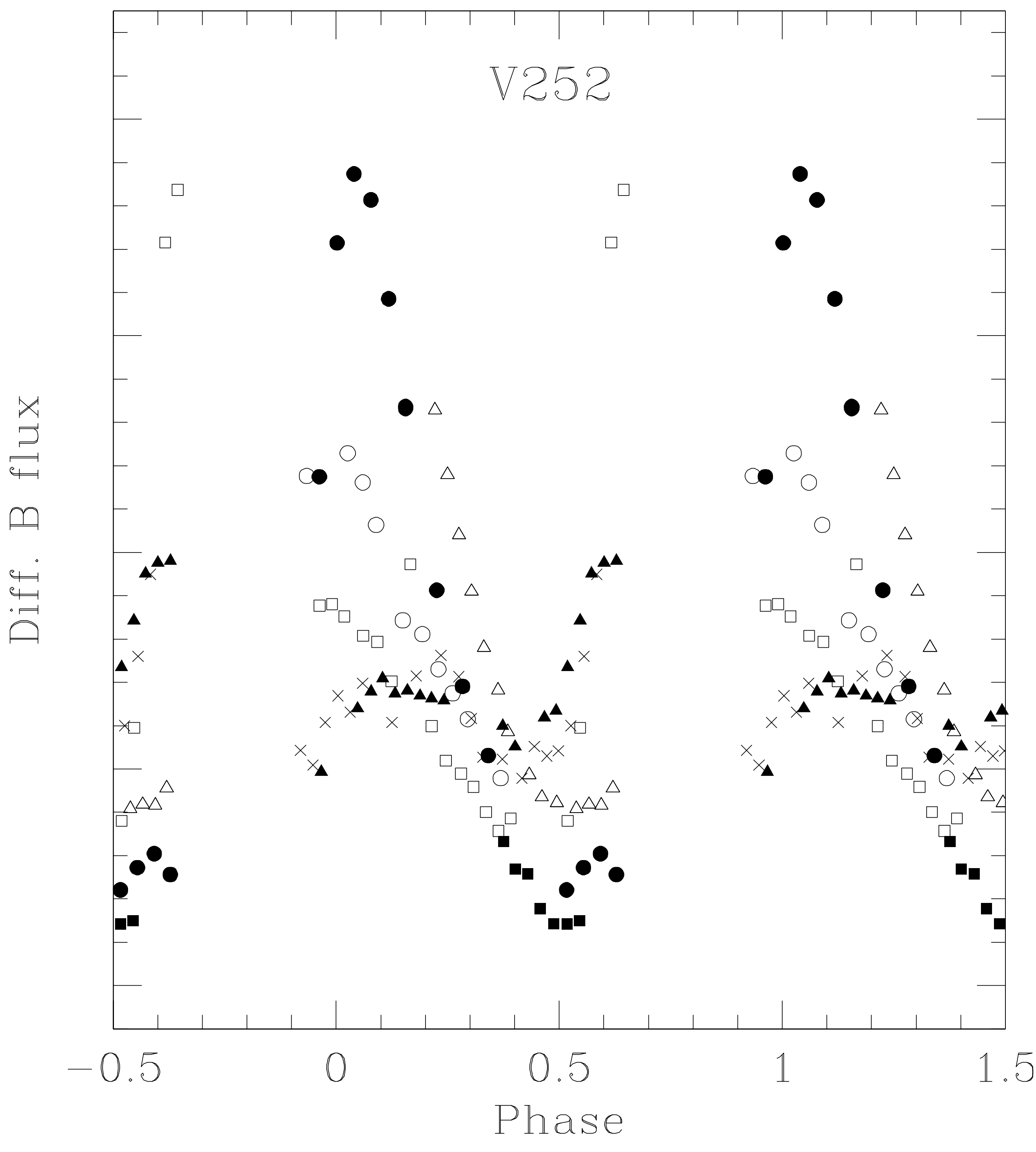


V251
P$_0$=0.47423 P$_1$=0.35384
E=2448756.820
−40.0
−20.0
0.0
20.0
ΔFlux
(Arb. Units)
0.0
0.2
0.4
0.6
0.8
1.0
Phase


NGC2419
NGC6426
M3
M3 new RRd's
IC4499
M68
M15
Milky Way-field
Sagittarius
LMC-field
LMC-clusters
SMC
Draco
Sculptor
Z=10⁻⁴: 0.85,0.80,0.75,0.65 M⊙
Z=2x10⁻⁴: 0.85,0.75,0.65 M⊙
Z=4x10⁻⁴: 0.75,0.65,0.60 M⊙
Z=6x10⁻⁴: 0.80,0.75,0.65 M⊙
Z=8x10⁻⁴: 0.75,0.65 M⊙
0.755
0.750
0.745
0.740
P1/P0
0.45
0.50
0.55
0.60
P0 (days)



0.748
0.746
0.744
0.742
0.740
0.738
0.40
0.45
0.50
0.55
P1/P0
P0 (days)
M3
M3 new RRd's
Z=10-4
Z=2x10-4
Z=4x10-4
Z=6x10-4
Z=8x10-4
0.80/1.72
0.85/1.72
0.80/1.72
0.75/1.72
0.75/1.72
0.75/1.72
0.65/1.61
0.60/1.65
0.65/1.72
0.75/1.72
0.75/1.72
0.65/1.65
0.65/1.72
0.65/1.72


V252
Diff. B flux
−0.5
0
0.5
1
1.5
Phase